Thermal Unit Commitment Including Optimal AC Power Flow Constraints

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Abstract

We propose a new algorithm for unit commitment that employs a Lagrange relaxation technique with a new augmentation of the Lagrangian. The new augmentation involves a duplication of variables that allows relaxation of the coupling between generator time-spanning constraints and system-wide instantaneous constraints. This framework allows the possibility of committing units that are required for the VArs that they can produce, as well as for their real power. Furthermore, although the algorithm is very CPU-intensive, the separation structure of the Lagrangian allows its implementation in parallel computers. Our work builds upon that of Batut & Renaud, as well as that of Baldick.

1 Introduction

Lagrangian relaxation as a technique for unit commitment has come a long way since it was first introduced, but there has been one constant central theme all along, namely, that of *separability*. Since the early papers [1, 3] this decomposability was the sought-after quality, and for a good reason: the unit commitment problem, being of a mixed-integer nature, suffers from combinatoric complexity as the number of generators increases. It is this feature that dooms other algorithms intended for solving it, such as dynamic programming: the combined state space of several generators in a dynamic program has a size that is too large to be able to tackle many realistic problems, even with limited–memory schemes. And it only gets worse as other constraints that increase the required state space (such as limited ramp rates) are introduced.

Lagrangian relaxation permits the decomposition of the problem into several one machine problems at each iteration; the coupling to other constraints involving more machines is achieved by sharing price information that is updated from one iteration to another. The complexity of a given iteration becomes linear in the number of generators instead of geometric. This property is what has given the technique an increased acceptance when compared to other techniques such as dynamic programming and branch and bound algorithms.

Mathematically, the unit commitment problem can be formulated as:

$$\min_{P,Q,U} \{ \; F(P,U) + K(U) \quad | \quad (P,U) \in \mathcal{D}, \;\; (P,Q,U) \in \mathcal{S}, \;\; (P,Q,U) \in \mathcal{C} \; \} \tag{1}$$

where

n_t:	Length of the planning horizon
n_g:	Number of generators to schedule
$p^{i,t}$:	Real power output for generator i at time t
$q^{i,t}$:	Reactive power output for generator i at time t
$u^{i,t}$:	On/off status (one or zero) for generator i at time t
P:	$(p^{i,t})$, $i = 1 \ldots n_g$, $t = 1 \ldots n_t$
Q:	$(q^{i,t})$, $i = 1 \ldots n_g$, $t = 1 \ldots n_t$
U:	$(u^{i,t})$, $i = 1 \ldots n_g$, $t = 1 \ldots n_t$
$F(P,U)$:	The total production cost
$K(U)$:	The sum of any startup costs
$\mathcal{D}$:	A set of *dynamic* generator-wise constraints
$\mathcal{S}$:	A set of *static* instantaneous constraints
$\mathcal{C}$:	A set of *nonseparable* constraints

It is assumed that the production cost function F is separable over each generator and time period so that $F(P,U) = \sum_{t=1}^{n_t} \sum_{i=1}^{n_g} u^{i,t} F^i(p^{i,t})$. For our purposes, the constraints of the problem have been separated into three kinds: The set $\mathcal{D}$ contains constraints that pertain to a single generator, but may span several time periods. These include minimum up or down times and ramping constraints. The set $\mathcal{S}$ contains constraints that span the complete system but involve only one time period, such as load/demand matching, voltage limits, reserve constraints and generation upper/lower limits. Finally, $\mathcal{C}$ is the set of constraints that involve more than one generator and more than one time period. A typical example is the infeasibility of turning on more than one unit at a time in a given location because of crew constraints.

Muckstadt and Koenig [3] introduced a first version of Lagrangian relaxation for the unit commitment problem. They considered a lumped one-node network with losses modeled as a fixed penalty factors. Reserve constraints were also considered. To illustrate the separation structure, we write an example formulation including demand and reserve constraints. Their relaxation yields a Lagrangian

$$\begin{aligned}\mathcal{L}(P,U,\lambda,\beta) &= F(P,U) + \sum_{t=1}^{n_t} \lambda^t (P_D^t - \sum_{i=1}^{n_g} u^{i,t} p^{i,t}) \\ &\quad + \sum_{t=1}^{n_t} \beta^t (R^t - \sum_{i=1}^{n_g} u^{i,t} P_{max}^i) \qquad (2)\end{aligned}$$

where P_D^t is the real power demand in period t and R^t is the desired minimum total commited capacity for the same period. One can then consider the dual objective

$$q(\lambda,\beta) = \min_{P,U} \ \mathcal{L}(P,U,\lambda,\beta) \qquad (3)$$

and corresponding dual problem

$$\max_{\lambda\geq 0,\beta\geq 0} q(\lambda,\beta) \qquad (4)$$

which can be written explicitly in the following form after collecting terms on a per generator basis

$$\max_{\lambda\geq 0,\beta\geq 0} \left\{ \sum_{t=1}^{n_t} (\lambda^t P_D^t + \beta^t R^t) \ + \min_{P,U} \left[\sum_{i=1}^{n_g} \sum_{t=1}^{n_t} (u^{i,t} F^i(p^{i,t}) - \lambda^t u^{i,t} p^{i,t} - \beta^t u^{i,t} P_{max}^i) \right] \right\} \qquad (5)$$

Thus, for fixed λ and β, finding the value of $q(\lambda,\beta)$ amounts to solving n_g separate, single-generator dynamic programs of the form

$$\min_{p^{i,\cdot},u^{i,\cdot}} \sum_{t=1}^{n_t} (u^{i,t} F^i(p^{i,t}) - \lambda^t u^{i,t} p^{i,t} - \beta^t u^{i,t} P_{max}^i) \qquad (6)$$

These dynamic programs can actually accomodate any $\mathcal{D}$–type constraint such as minimal up or down times, as well as any startup costs. Ramp-rate constraints can also be introduced by discretizing the generation range for the unit, although the size of the state space grows considerably. For a detailed description of a dynamic programming graph including most of these constraints, see reference [5].

This suggests that a dual maximization algorithm is better suited to this particular problem because it can exploit the separation structure of the dual objective. A subgradient–based dual maximization algorithm applied to the unit commitment problem proceeds as follows:

Algorithm 1: *Classical Lagrangian relaxation*

Step 0 $k \leftarrow 0$

Step 1 Initialize λ_k and β_k to a sensible (under estimate) value

Step 2 Compute

$$(\hat{P}_k, \hat{U}_k) \longleftarrow \arg \min_{\text{feasible } P,U} \mathcal{L}(P,U,\lambda_k,\beta_k)$$

by solving n_g single–generator dynamic programs that incorporate any $\mathcal{D}$–type constraints and any startup costs.

Step 3 The dual cost is $q(\lambda_k,\beta_k) = \mathcal{L}(\hat{P}_k,\hat{U}_k,\lambda_k,\beta_k)$

Step 4 The primal cost is infinite if the schedule $\hat{U}_k$ is infeasible; else it is the cost at the solution of

$$\min_{\text{feasible } P} F(P,\hat{U}_k)$$

where P feasible means that it satisfies the demand. This problem is separable into n_t economic dispatches. If there are any startup costs, they should be added too.

Step 5 Compute the duality gap at this iteration as *primal cost – dual cost*

Step 6 If the gap is small enough, stop; else, update λ, β according to a subgradient maximization technique, for example Poljak's formula:

$$\begin{aligned}\lambda_{k+1}^t &\leftarrow \lambda_k^t + \frac{1}{\alpha_0 + \alpha_1 k} (P_D^t - \sum_{i=1}^{n_g} \hat{u}^{i,t} \hat{p}^{i,t}) \\ \beta_{k+1}^t &\leftarrow \beta_k^t + \frac{1}{\alpha_0 + \alpha_1 k} (R^t - \sum_{i=1}^{n_g} \hat{u}^{i,t} P_{max}^i)\end{aligned}$$

Step 7 $k \leftarrow k+1$; Go to Step 2.

Such is the basic idea behind Lagrangian relaxation. In the past 20 years, advances have been made in several areas, enhancing the number and type of constraints that can be treated, addressing some convergence issues when the cost is not strongly convex, and so on. In 1983, Bertsekas et. al. [6] described an algorithm that included many refinements in the dynamic programming subproblem, as well as proof that the expected relative duality gap is inversely proportional to the number of generators; this was good news for large–scale problems. Also in 1983, Merlin and Sandrin [7] reported a Lagrangian relaxation method with linear costs, reserve constraints, exponential restart costs (but not banking capabilities) and special λ–updates that take into account the kind of constraints that are violated and some properties of linear cost functions. In 1988, Zhuang and Galiana [9] reported a three–stage method involving (1) Standard Lagrangian relaxation without reserve constraint, (2) A reserve feasibility search, and (3) An economic dispatch stage. At the time, several methodologies for achieving reserve feasibility were being tested. Most relied on further stepping up the multipliers for the demand constraints, thus increasing the number of commited generators. At issue was whether to raise all multipliers simultaneously or sequentially, starting with those of time periods where the reserve constraint was most unfulfilled. Reserve feasibility search has been an active area and the difficulty is especially important in so–called *indirect methods*[1]. It is in part due to the fact that a dual solution does not necessarily meet the primal's constraints. Of course, if those constraints were not in the dual problem in the first place, primal feasibility is even more of an issue.

Sophisticated as the schemes were becoming, the underlying network was being largely ignored. In [11], Ružić and Rajaković include transmission line transfer limits using a DC flow model and transmission losses using constant factors. This can be done because in the Lagrangian such relaxed constraints are linear in $u^{i,t}p^{i,t}$ and $u^{i,t}$, so it is still possible to collect all terms on a per-generator basis, achieving separation into n_g dynamic programs. However, even with just two congested lines the computation times escalated. This seems to be inherent to binding constraints in subgradient methods, especially if poorly scaled.

Several other papers have followed the DC flow formulation in their incorporation of line limits to the dual maximization, including [13, 15, 16, 17, 18]. Baldick [15] uses a general formulation that could in principle be used to address AC flow constraints, but the specific algorithm that he describes still uses a basic DC flow approximation. It seems that the general rule of thumb is: if a constraint is linear, then add it to the Lagrangian, appropriately relaxed with a multiplier, and separation will be preserved. As a matter of fact, any constraint $g(P,U) = \sum_{t=1}^{n_t} \sum_{i=1}^{n_g} g^{i,t}(u^{i,t}, p^{i,t})$ can be addressed in the dual maximization while preserving separation. Others have followed this trend of addressing more and more linear constraints in the dual optimization. For example, in [12], ramping constraints are relaxed as well, so that the dynamic programs do not have to deal explicitly with ramp constraints, but $2n_tn_g$ more multipliers are needed. This idea is also used in [20].

There are several possible drawbacks to this overall scheme of adding more linear constraints to the formulation and dealing with them in the dual optimization phase. The first one is that the number of dual variables grows very large. In general, this does not seem to be a problem with regards to convergence, unless many of the constraints that they represent are actually binding. However, it does increase the amount of memory needed: for line limit multipliers, for example, $2n_ln_t$ variables may be needed, n_l being the number of lines in the network.

The second drawback applies to only some types of constraints, such as line limits modeled by means of DC flow sensitivities: they are not sparse, although one could conceivably zero out small elements. This does not apply to inherently sparse constraints such as ramp rate limits, but hinders the scalability of the DC flow approach to incorporation of line limits to the dual optimization phase, especially when considering line outages that are valid only for some time periods; this would make it necessary to consider several sets of sensitivities. Furthermore, the DC flow is just an approximation that may be significantly off in some cases.

A third (and more important) drawback is that some potentially important constraints cannot be formulated as linear. For example, consider the case of voltage limits, where it is necessary to perform a power flow to investigate their values. However, one should notice that, complicated as AC OPF constraints are, they still fall neatly into the category of $\mathcal{S}$–type constraints: they apply to all generators, but only at one time period. We shall take advantage of this in the following section.

[1] In reference [17], Shaw distinguishes between *direct* and *indirect* methods, where the former address reserve feasibility and other OPF constraints in the dual optimization phase, whereas the latter deal with such constraints only after having generated a commitment schedule with a dual maximization that did not include such constraints and thus make *post factum* corrections.

2 Unit commitment with AC OPF formulation

Our approach has its roots in the *variable duplication* technique credited to Guy Cohen in [13] by Batut and Renaud. This same technique was used later by Baldick [15] in his more general formulation of the unit commitment problem. The main technical achievement of our paper is the inclusion of reactive power output variables to the formulation, so that better loss management may be performed and generators that are necessary because of their VAr output but not their real power are actually commited. This is the logical next step in the development of Lagrangian relaxation techniques. At this point, when typical algorithms reduce the duality gap to figures close to 1%, it is important to recognize that a better handling of the reactive power considerations at the unit commitment stage may have a payoff that is higher than those few last percentage points in the duality gap.

We start by defining two sets of variables, the *dynamic variables* and the *static* ones:

Dynamic:

- $u^{i,t}$: Commitment status $\{0,1\}$ for generator i at time t
- $d_p^{i,t}$: Real power output for generator i at time t
- $d_q^{i,t}$: VAr output for generator i at time t
- U: $(u^{i,t})$, $i=1\ldots n_g$, $t=1\ldots n_t$
- D_p: $(d_p^{i,t})$, $i=1\ldots n_g$, $t=1\ldots n_t$
- D_q: $(d_q^{i,t})$, $i=1\ldots n_g$, $t=1\ldots n_t$
- D: (D_p, D_q)

Static:

- $s_p^{i,t}$: Real power output for generator i at time t
- $s_q^{i,t}$: VAr output for generator i at time t
- S_p: $(s_p^{i,t})$, $i=1\ldots n_g$, $t=1\ldots n_t$
- S_q: $(s_q^{i,t})$, $i=1\ldots n_g$, $t=1\ldots n_t$
- S: (S_p, S_q)

Then the following optimization problem is defined

$$\min_{D,U,S} \sum_{t=1}^{n_t}\sum_{i=1}^{n_g}[u^{i,t}F^i(d_p^{i,t}) + K^{i,t}(u^{i,\cdot})] \qquad (7)$$

subject to:

(1) $\mathcal{D}$–type constraints

$$P^i_{min} \le d_p^{i,t} \le P^i_{max}, \qquad (8)$$

$$Q^i_{min} \le d_q^{i,t} \le Q^i_{max}, \qquad (9)$$

$$U \text{ satisfies minimal up and down times}, \qquad (10)$$

(2) $\mathcal{S}$–type constraints

$$0 \le s_p^{i,t} \le P^i_{max}, \qquad (11)$$

$$Q^i_{min} \le d_q^{i,t} \le Q^i_{max}, \qquad (12)$$

$$(S_p, S_q) \left\{ \begin{array}{l} \text{satisfies the network load flow} \\ \text{equations while respecting line} \\ \text{MVA limits and voltage limits} \end{array} \right. \qquad (13)$$

(3) and the following additional constraints

$$R^{l,t} - \sum_{i\in Z_l} u^{i,t}P^i_{max} \le 0, \quad l = 1\ldots n_z, \quad t = 1\ldots n_t \qquad (14)$$

$$s_p^{i,t} - u^{i,t}d_p^{i,t} = 0, \quad i = 1\ldots n_g, \quad t = 1\ldots n_t \qquad (15)$$

$$s_q^{i,t} - u^{i,t}d_q^{i,t} = 0, \quad i = 1\ldots n_g, \quad t = 1\ldots n_t \qquad (16)$$

where $R^{l,t}$ is the minimum combined capacity that is acceptable for the lth zone in the tth period and Z_l is the set of indices of generators in the lth zone.

We will assume that we can enforce both the $\mathcal{D}$ constraints (8–10) and the $\mathcal{S}$ constraints (11–13), so that we only relax the three last constraints (14–16), which leads to the following Lagrangian:

$$\begin{aligned}
\mathcal{L}(U,D,\lambda,\beta) \ = & \\
& \sum_{t=1}^{n_t}\sum_{i=1}^{n_g}[u^{i,t}F^i(d_p^{i,t}) + K^{i,t}(u^{i,\cdot})] \\
& + \sum_{t=1}^{n_t}\sum_{l=1}^{n_z}\beta^{l,t}(R^{l,t} - \sum_{i\in Z_l} u^{i,t}P^i_{max}) \\
& + \sum_{t=1}^{n_t}\sum_{i=1}^{n_g}\lambda_p^{i,t}(s_p^{i,t} - u^{i,t}d_p^{i,t}) \\
& + \sum_{t=1}^{n_t}\sum_{i=1}^{n_g}\lambda_q^{i,t}(s_q^{i,t} - u^{i,t}d_q^{i,t}) \qquad (17) \\
= & \sum_{i=1}^{n_g}\sum_{t=1}^{n_t}\Big\{u^{i,t}F^i(d_p^{i,t}) + K^{i,t}(u^{i,\cdot}) - \lambda_p^{i,t}u^{i,t}d_p^{i,t} \\
& - \beta^{z(i),t}u^{i,t}P^i_{max} - \lambda_q^{i,t}u^{i,t}d_q^{i,t}\Big\} \\
& + \sum_{t=1}^{n_t}\sum_{i=1}^{n_g}(\lambda_p^{i,t}s_p^{i,t} + \lambda_q^{i,t}s_q^{i,t}) \\
& + \sum_{t=1}^{n_t}\sum_{l=1}^{n_z}\beta^{l,t}R^{l,t} \qquad (18) \\
= & \ \mathcal{L}_1(U,D,\lambda,\beta) + \mathcal{L}_2(S,\lambda) + \mathcal{L}_3(\beta) \qquad (19)
\end{aligned}$$

where $\lambda = (\lambda_p^{i,t}, \lambda_q^{i,t})$ are multipliers on the relaxed equalities of the two kinds of variables, $\beta^{l,t}$ is the multiplier associated to the lth zone's reserve requirement at the tth period, and $z(i)$ returns the index of the zone to which generator i belongs.

The separation structure of the Lagrangian is obvious upon looking at equations (18) and (19). It makes it possible to write the dual objective as

$$\begin{aligned} q(\lambda,\beta) &= \min_{U,D,S}\{\mathcal{L}_1(U,D,\lambda,\beta)+\mathcal{L}_2(S,\lambda)+\mathcal{L}_3(\beta)\} \\ &= \min_{U,D}\mathcal{L}_1(U,D,\lambda,\beta) \\ &\quad + \min_{S}\mathcal{L}_2(S,\lambda) \\ &\quad + \mathcal{L}_3(\beta) \end{aligned} \tag{20}$$

By looking again at (18) and (20), it can be seen that the first term can be computed by solving n_g dynamic programs again; the second term separates into n_t optimal power flow problems with all generators commited but with special cost curves $\lambda_p^{i,t}s_p^{i,t} + \lambda_q^{i,t}s_q^{i,t}$ for generator i at time t. Notice that $s_q^{i,t}$ also has a price. It is assumed that the solutions of the dynamic programs meet the $\mathcal{D}$ constraints and that the solutions of the optimal power flows meet the $\mathcal{S}$ constraints.

It would be tempting to apply a dual maximization procedure to the dual objective as stated, but there are some issues that prevent us from doing that without some modification of the Lagrangian. The first issue is that the cost of $d_q^{i,t}$ reflected in the dynamic programs, being linear, is not strongly convex; this can cause unwanted oscillations in the $d_q^{i,t}$ prescribed by the dynamic program (see [13]). Therefore we set out to fix this before addressing any other problems by augmenting the Lagrangian with quadratic functions of the equality constraints. This will introduce nonseparable terms, but using the Auxiliary Problem Principle described by G. Cohen in [4] and [8] we can linearize those terms about the previous iteration values, rendering them separable. Thus we write the new augmented Lagrangian as

$$\begin{aligned} \mathcal{L}(U,D,S,\lambda,\beta) \;=\; & \\ & \sum_{t=1}^{n_t}\sum_{i=1}^{n_g}[u^{i,t}F^i(d_p^{i,t}) + K^{i,t}(u^{i,\cdot})] \\ & + \sum_{t=1}^{n_t}\sum_{l=1}^{n_z}\beta^{l,t}(R^{l,t} - \sum_{i\in Z_l}u^{i,t}P_{max}^i) \\ & + \sum_{t=1}^{n_t}\sum_{i=1}^{n_g}\lambda_p^{i,t}(s_p^{i,t} - u^{i,t}d_p^{i,t}) \\ & + \sum_{t=1}^{n_t}\sum_{i=1}^{n_g}\lambda_q^{i,t}(s_q^{i,t} - u^{i,t}d_q^{i,t}) \\ & + \sum_{t=1}^{n_t}\sum_{i=1}^{n_g}\frac{c_p}{2}(s_p^{i,t} - u^{i,t}d_p^{i,t})^2 \\ & + \sum_{t=1}^{n_t}\sum_{i=1}^{n_g}\frac{c_q}{2}(s_q^{i,t} - u^{i,t}d_q^{i,t})^2 \end{aligned} \tag{21}$$

The Auxiliary Problem Principle allows us to substitute the augmentation terms by the following at iteration k (see [13])

$$\begin{aligned} & \sum_{t=1}^{n_t}\sum_{i=1}^{n_g}c_p(\bar{s}_p^{i,t} - \bar{u}_p^{i,t}\bar{d}_p^{i,t})(s_p^{i,t} - u^{i,t}d_p^{i,t}) \\ & + \sum_{t=1}^{n_t}\sum_{i=1}^{n_g}\frac{b_p}{2}\left\{(s_p^{i,t} - \bar{s}_p^{i,t})^2 + (u^{i,t}d_p^{i,t} - \bar{u}^{i,t}\bar{d}_p^{i,t})^2\right\} \\ & + \sum_{t=1}^{n_t}\sum_{i=1}^{n_g}c_q(\bar{s}_q^{i,t} - \bar{u}_q^{i,t}\bar{d}_q^{i,t})(s_q^{i,t} - u^{i,t}d_q^{i,t}) \\ & + \sum_{t=1}^{n_t}\sum_{i=1}^{n_g}\frac{b_q}{2}\left\{(s_q^{i,t} - \bar{s}_q^{i,t})^2 + (u^{i,t}d_q^{i,t} - \bar{u}^{i,t}\bar{d}_q^{i,t})^2\right\} \end{aligned} \tag{22}$$

where $\bar{u}^{i,t}$, $\bar{d}_p^{i,t}$, $\bar{d}_q^{i,t}$, $\bar{s}_p^{i,t}$ and $\bar{s}_q^{i,t}$ are the values obtained at the $(k-1)$th iteration. Since (22) is separable, we can collect terms of the augmented Lagrangian on a per-generator basis, so that at the kth iteration we are faced with

$$\begin{aligned} & \mathcal{L}(U,D,S,\lambda,\beta,\bar{U},\bar{D},\bar{S}) = \\ & \quad \sum_{i=1}^{n_g}\sum_{t=1}^{n_t}\Big\{u^{i,t}F^i(d_p^{i,t}) + K^{i,t}(u^{i,\cdot}) \\ & \qquad + \frac{b_p}{2}u^{i,t}(d_p^{i,t})^2 + \frac{b_q}{2}u^{i,t}(d_q^{i,t})^2 \\ & \qquad + \left[-\lambda_p^{i,t} - c_p(\bar{s}_p^{i,t} - \bar{u}^{i,t}\bar{d}_p^{i,t}) - b_p\bar{u}^{i,t}\bar{d}_p^{i,t}\right]u^{i,t}d_p^{i,t} \\ & \qquad + \left[-\lambda_q^{i,t} - c_q(\bar{s}_q^{i,t} - \bar{u}^{i,t}\bar{d}_q^{i,t}) - b_q\bar{u}^{i,t}\bar{d}_q^{i,t}\right]u^{i,t}d_q^{i,t} \\ & \qquad + \left[-\beta^{z(i),t}P_{max}^i\right]u^{i,t}\Big\} \\ & \quad + \sum_{t=1}^{n_t}\sum_{i=1}^{n_g}\Big\{\frac{b_p}{2}(s_p^{i,t})^2 + \frac{b_q}{2}(s_q^{i,t})^2 \\ & \qquad + \left[\lambda_p^{i,t} + c_p(\bar{s}_p^{i,t} - \bar{u}^{i,t}\bar{d}_p^{i,t}) - b_p\bar{s}_p^{i,t}\right]s_p^{i,t} \\ & \qquad + \left[\lambda_q^{i,t} + c_q(\bar{s}_q^{i,t} - \bar{u}^{i,t}\bar{d}_q^{i,t}) - b_q\bar{s}_q^{i,t}\right]s_q^{i,t}\Big\} \\ & \quad + \sum_{t=1}^{n_t}\sum_{i=1}^{n_g}\Big\{\frac{b_p}{2}\left[(\bar{s}_p^{i,t})^2 + (\bar{u}^{i,t}\bar{d}_p^{i,t})^2\right] \\ & \qquad + \frac{b_q}{2}\left[(\bar{s}_q^{i,t})^2 + (\bar{u}^{i,t}\bar{d}_q^{i,t})^2\right]\Big\} \\ & \quad + \sum_{t=1}^{n_t}\sum_{l=1}^{n_z}\beta^{l,t}R^{l,t} \qquad (23) \\ & \quad = \mathcal{L}_1(U,D,\lambda,\beta,\bar{U},\bar{D},\bar{S}) + \mathcal{L}_2(S,\lambda,\bar{U},\bar{D},\bar{S}) \\ & \qquad + \mathcal{L}_3(\beta) \qquad (24) \end{aligned}$$

Notice that (24) has the same separation structure of (19).

Now that the separability issue has been resolved, we propose the following

Algorithm 2: *AC Augmented Lagrangian relaxation*

Step 0 $k \longleftarrow 0$

Step 1 Initialize $(\lambda_p^{i,t}, \lambda_q^{i,t})$ to the values of the multipliers on the power flow equality constraints at generator buses when running an OPF with all units commited. Initialize $(\bar{U}, \bar{D}, \bar{S})$ to zeros.

Step 2a Compute

$$(\hat{U}, \hat{D}) \leftarrow \arg \min_{\text{feasible } U,D} \mathcal{L}_1(U, D, \lambda, \beta, \bar{U}, \bar{D}, \bar{S})$$

by solving n_g one-generator dynamic programs.

Step 2b Compute

$$\hat{S} \longleftarrow \arg \min_{\text{feasible } S} \mathcal{L}_2(S, \lambda, \bar{U}, \bar{D}, \bar{S})$$

by solving n_t OPF's in which all generators are committed, their generation range has been expanded to include $P^i_{min} = 0$ and the special cost $\mathcal{L}_2(S, \lambda, \bar{U}, \bar{D}, \bar{S})$ is used. Note: all tasks in steps 2a and 2b can be solved in parallel.

Step 3 If the commitment schedule $\hat{U}$ is not in a database of tested commitments, perform a cheap primal feasibility test. If the results are not encouraging, store the schedule in the database and label it "infeasible", then go to Step 6.

Step 4 Perform a more serious primal feasibility test by actually attempting to run n_t OPF's with the original P_{min} constraints. If all OPF's are successful, store the commitment in the database, together with the primal cost including startup costs, and the duality gap (the dual cost was available upon solving 2a and 2b). Else label the commitment as "infeasible", store it in the database, and go to Step 6.

Step 5 If the duality gap is small enough, stop.

Step 6 Update all multipliers using subgradient techniques, and

$$\begin{aligned} \bar{U} &\longleftarrow \hat{U} \\ \bar{D} &\longleftarrow \hat{D} \\ \bar{S} &\longleftarrow \hat{S} \\ k &\longleftarrow k+1 \end{aligned}$$

Step 7 Go to Step 2.

The proposed algorithm is very OPF–intensive: the major computational cost is that of computing n_t OPF's for every iteration in order to solve the static subproblems, plus extra OPF's in selected iterations when a given commitment is promising. Thus, every effort possible must be made to try to alleviate the burden of OPF computation. The first thing that can be done is to use as a starting point for the OPF the result of the previous iteration for the same time period. Most of the times, the only difference in the data for the OPF would be a small change in the costs (reflected by the change in λ from one iteration to another). This should result in fewer iterations needed for the OPF.

Another drawback of the algorithm is that a different set of OPF computations must be performed to compute the value of the dual objective and to compute the value of the primal. Thus, before even trying to compute the value of the primal objective, one should make sure that such a costly computation is worth doing. Some of the cheap tests include verifying that the reserve constraint is met and that the mismatch between the S and the D variables is small. With respecto to the latter, we have found that if $u^{1,t} = 1$, a smaller mismatch should be asked for as requisite to feasibility than if $u^{i,t} = 0$. More costly feasibility tests would involve power flow problems starting from appropriate initial values. Finally, since Alsac *et al.* [10] claim that LP–based OPF methods can be faster in detecting infeasibility, it might be advantageous to use such methods.

3 Preliminary computational results

We have written a preliminary implementation of the algorithm in the *MATLAB*™ environment. The dynamic subproblems can accomodate minimal up or down times, warm start and cold startup costs. The static subproblems are solved by an OPF code (see [22]) that incorporates box constraints on generator's P and Q, polynomial cost functions for both P and Q, voltage constraints, line MVA limits and of course, the power flow equations. The program has been tested on a modified IEEE 30-bus system [2] with 6 generators and a planning horizon of length 6. For comparison purposes, a version of the Lagrangian relaxation algorithm with DC Flow-based relaxed line limits was also written. It turns out that generator number 4, located at bus number 27, is needed for voltage support for many load levels even though it is most uneconomical to operate. The AC-based algorithm correctly identified this unit as a must-run for those time periods, even providing some price information on the MVArs that this unit produced by means of the corresponding $\lambda_q^{i,t}$. The number of iterations required was usually in the vicinity of one hundred. In contrast, the DC flow-based algorithm failed to commit unit 4 for any period, producing a commitment schedule that was infeasible in light of the AC power flow constraints.

The importance of proper selection of the (c_p, b_p, c_q, b_q) parameters was apparent from the beginning. We obtained good results with $c_p = 0.05$, $b_p = 4c_p$, $c_q = 0.08$ and $b_q = 4c_q$. However, other choices tended to produce somewhat smooth, damped oscillations in the values of some of the $(\lambda_p^{i,t}, \lambda_q^{i,t})$.

To highlight one of the new features found in the algorithm, we show the evolution of $(\lambda_p^{i,t}, \lambda_q^{i,t})$ vs. iteration number for a typical run in figure 1. The multipliers with the higher values are all P–type multipliers. Those with the smaller values correspond to the $\lambda_q^{i,t}$. Most of them settle to zero, indicating that Q is essentially free almost always. However, a few of them actually have high prices: these belong to generators and time periods where the OPF tries to use their MVArs in order to force feasibility or guided by economic considerations, but the generators are not actually committed. In the course of the algorithm, these $\lambda_q^{i,t}$ may grow so large that they trigger the respective unit on. Once this happens, such multipliers tend to approach zero again, since Q is now plentiful. In figure 1 there are two clear examples of this behavior, corresponding to unit 4 being commited for certain time periods. As the multiplier approaches zero, the static copy $s_q^{i,t}$ will approach the dynamic $d_q^{i,t}$.

4 Future work

At the time that this paper was written, the implementation served the purpose of testing the overall algorithm's expected behavior. The results that were obtained encourage us to believe that the formulation is sound. However, clearly more work is needed in order to produce anything close to practical. More kinds of constraints (e.g., ramp limits) need to be included in the implementation. Preparations for testing larger scale systems are under way, and, if successful, a parallel implementation will be worth pursuing. At the conference, we should have more experience with the algorithm and more complete data to report. We conclude this paper with the following comment: since computer capacity grows much faster than the size of the electrical power systems in the world, we believe that this algorithm or a variant of it could well be solving real life unit commitment problems in a few years.

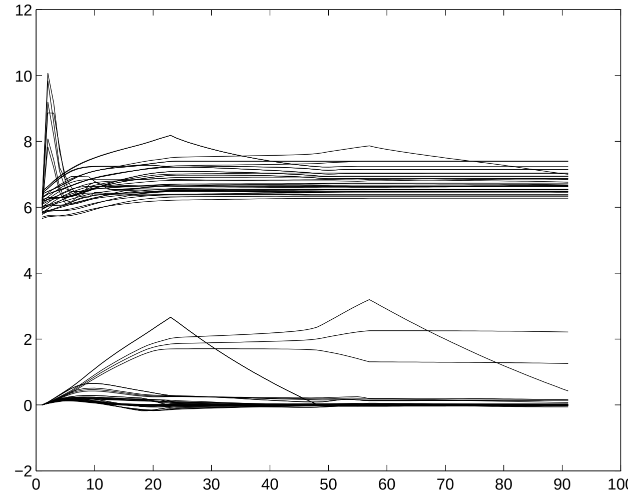

Figure 1: Evolution of multipliers in a typical run.

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References

[1] Muckstadt, J.A. & Wilson, R.C., "An Application of Mixed-Integer Programming Duality to Scheduling Thermal Generating Systems", *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-87, No. 12, December 1968, pp. 1968–1978.

[2] Alsac, O. & Stott, B., "Optimal Load Flow with Steady-State Security", *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS 93, No. 3, 1974, pp. 745–751.

[3] Muckstadt, J.A. & Koenig, S.A., "An Application of Lagrange Relaxation to Scheduling in Power–Generation Systems", *Operations Research*, Vol. 25, No. 3, May–June 1977, pp. 387–403.

[4] Cohen, G., "Auxiliary Problem Principle and Decomposition of Optimization Problems", *Journal of Optimization Theory and Applications*, Vol. 32, No. 3, November 1980, pp. 277–305.

[5] Lauer, G.S., Bertsekas, D.P., Sandell, N.R. & Posbergh, T.A., "Solution of Large–Scale Optimal Unit Commitment Problems", *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-101, No. 1, January 1982, pp. 79–85.

[6] Bertsekas, D.P., Lauer, G.S., Sandell, N.R. & Posbergh, T.A., "Optimal Short-Term Scheduling of Large-Scale Power Systems", *IEEE Transactions on Automatic Control*, Vol. AC-28, No. 1, January 1983, pp. 1–11.

[7] Merlin, A. & Sandrin, P., "A New Method for Unit Commitment at Electricité de France", *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-102, No. 5, May 1983, pp. 1218–1225.

[8] Cohen, G. & Zhu, D.L., "Decomposition Coordination Methods in Large Scale Optimization Problems: The Nondifferentiable Case and the Use of Augmented Lagrangians", in *Advances in Large Scale Systems, Vol. 1; J.B. Cruz, Ed.*, JAI Press Inc, 1984, pp. 203–266.

[9] Zhuang, F. & Galiana, F.D., "Towards a More Rigorous and Practical Unit Commitment by Lagrangian Relaxation", *IEEE Transactions on Power Systems*, Vol. 3, No. 2, May 1988, pp. 763–773.

[10] Alsac, O., Bright, J., Prais, M. & Stott, B., "Further Development in LP–Based Optimal Power Flow", *IEEE Transactions on Power Systems*, Vol. 5, No. 3, August 1990, pp. 697–711.

[11] Ružić, S. & Rajaković, N., "A New Approach for Solving Extended Unit Commitment Problem", *IEEE Transactions on Power Systems*, Vol. 6, No. 1, February 1991, pp. 269–277.

[12] Guan, X., Luh, P.B. & Yan, H., "An Optimization–Based method for Unit Commitment", *Electrical Power & Energy Systems*, Vol. 14, No. 1, February 1992.

[13] Batut, J. & Renaud, A., "Daily Generation Scheduling Optimization with Transmission Constraints: A New Class of Algorithms", IEEE Transactions on Power Systems, Vol. 7, No. 3, August 1992, pp. 982–989.

[14] Sheble, G.B. & Fahd, G.N., "Unit Commitment Literature Synopsis", *IEEE Transactions on Power Systems*, Vol. 9, No. 1, February 1994, pp. 128–135.

[15] Baldick, R., "The Generalized Unit Commitment Problem", *IEEE Transactions on Power Systems*, Vol. 10, No. 1, February 1995, pp. 465–475.

[16] Wang, S.J., Shahidehpour, S.M., Kirschen, D.S., Mokhtari, S. & Irisarri, G.D., "Short-Term Generation Scheduling with Transmission and Environmental Constraints Using an Augmented Lagragian Relaxation", *IEEE Transactions on Power Systems*, Vol. 10, No. 3, August 1995, pp. 1294–1301.

[17] Shaw, J.J., "A Direct Method for Security–Constrained Unit Commitment", *IEEE Transactions on Power Systems*, Vol. 10, No. 3, August 1995, pp. 1329–1339.

[18] Abdul-Rahman, K.H., Shahidehpour, S.M., Aganagic, M. & Mokhtari, S., "A Practical Resource Scheduling with OPF Constraints", *IEEE Transactions on Power Systems*, Vol. 11, No. 1, February 1996, pp. 254–259.

[19] Mészáros, Cs., " The efficient implementation of interior point methods for linear programming and their applications", Ph.D. Thesis, Eötvös Loránd University of Sciences, 1996.

[20] Svoboda, A.J., Tseng, C.L., Li, C.A. & Johnson, R.B., "Short-Term Resource Scheduling with Ramp Constraints", *IEEE Transactions on Power Systems*, Vol. 12, No. 1, February 1997, pp. 77–83.

[21] Li, C.A., Johnson, R.A. & Svoboda, A.J., "A New Unit Commitment Method", *IEEE Transactions on Power Systems*, Vol. 12, No. 1, February 1997, pp. 113–119.

[22] Zimmerman, R. & Gan, D., "*MATPOWER:* A Matlab Power System Simulation Package", http://www.pserc.cornell.edu/matpower/ .