Thermal Unit Commitment with Nonlinear Power Flow Constraints

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Abstract

A formulation of the thermal unit commitment problem including nonlinear power flow constraints is presented, making the use of more realistic constraint models possible. It also allows potential VAr production to be used as a criterion for the commitment of generators in strategic locations of the network. The Lagrangian Relaxation framework and a variable duplication technique are employed, permitting exploitation of the separation structure of the dual cost. Some results for small to medium-sized systems are reported.

Keywords: Thermal unit commitment, Power generation scheduling, Lagrangian relaxation.

1 Introduction

The thermal unit commitment problem, being of a mixed–integer nature, suffers from combinatorial complexity that is further compounded by the sheer size of real life problems of this kind. Indeed, without resorting to special techniques such as explotation of structure, the problem is basically unsolvable. For example, limited–memory dynamic programming schemes have been tried. Unfortunately, as the number of generators increases, the size of the combined state space for the dynamic programming problem grows combinatorially, and such schemes may easily fail to include the states that form part of the optimal solution in the dynamic programming graph.

Lagrangian relaxation is aimed towards the exploitation of the structure of the problem by achieving a specific form of separability in the the dual objective. First introduced by Muckstadt and Koenig in [2], Lagrangian relaxation permits decomposing the discrete on/off state variables into subsets belonging to each generator, essentially separating the combined discrete variable problem into several independent, one–generator dynamic programs. To coordinate the solution of the overall problem, a price sharing scheme is implemented. Hence, one trades the decomposition of the problem for having to do many single–generator iterations that share prices updated from iteration to iteration. The separability is possible thanks to the unique structure of the constraints. To illustrate, consider this basic formulation of the unit commitment problem:

$$\min_{P,Q,U} \{ \; F(P,U) + K(U) \quad | \quad (P,U) \in \mathcal{D}, \;\; (P,Q,U) \in \mathcal{S}, \;\; (P,Q,U) \in \mathcal{C} \; \} \quad (1)$$

where

n_t:	Length of the planning horizon
n_g:	Number of generators to schedule
$p^{i,t}$:	Real power output for generator i at time t
$q^{i,t}$:	Reactive power output for generator i at time t
$u^{i,t}$:	On/off status (one or zero) for generator i at time t
P:	$(p^{i,t})$, $i = 1 \ldots n_g$, $t = 1 \ldots n_t$
Q:	$(q^{i,t})$, $i = 1 \ldots n_g$, $t = 1 \ldots n_t$
U:	$(u^{i,t})$, $i = 1 \ldots n_g$, $t = 1 \ldots n_t$
$F(P,U)$:	The total production cost
$K(U)$:	The sum of any startup costs

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$\mathcal{D}$: A set of *dynamic* generator-wise constraints

$\mathcal{S}$: A set of *static* instantaneous constraints

$\mathcal{C}$: A set of *nonseparable* constraints

The production cost function F is separable over each generator and time period. The constraints of the problem have been classified into three kinds: Constraints that are related to a single generator (but could conceivably span several time periods) are lumped together in the set $\mathcal{D}$. Examples are minimum up or down times and ramping constraints. Constraints that span the complete system but involve only one time period, such as load/demand matching, voltage limits, reserve constraints and generation upper/lower limits, are classified as $\mathcal{S}$–type constraints. Finally, $\mathcal{C}$ is the set of constraints that involve more than one generator and more than one time period.

With this setup, the most basic example of the application of the Lagrangian relaxation proceeds as follows (see [2]): Assume that the $\mathcal{S}$ set involves a generation–meets–demand (no network effects) constraint and a reserve constraint for each time period. The relaxation of these constraints yields a Lagrangian

$$\mathcal{L}(P,U,\lambda,\beta) = F(P,U) + \sum_{t=1}^{n_t} \lambda^t (P_D^t - \sum_{i=1}^{n_g} u^{i,t} p^{i,t}) + \sum_{t=1}^{n_t} \beta^t (R^t - \sum_{i=1}^{n_g} u^{i,t} P_{max}^i) \quad (2)$$

where P_D^t is the real power demand in period t and R^t is the desired minimum total commited capacity for the same period. One can then consider the dual objective

$$q(\lambda,\beta) = \min_{P,U} \mathcal{L}(P,U,\lambda,\beta) \quad (3)$$

and corresponding dual problem

$$\max_{\lambda\geq 0,\beta\geq 0} q(\lambda,\beta) \quad (4)$$

which can be written explicitly in the following form after collecting terms on a per generator basis

$$\max_{\lambda\geq 0,\beta\geq 0} \left\{ \sum_{t=1}^{n_t} (\lambda^t P_D^t + \beta^t R^t) + \min_{P,U} \left[\sum_{i=1}^{n_g} \sum_{t=1}^{n_t} (u^{i,t} F^i(p^{i,t}) - \lambda^t u^{i,t} p^{i,t} - \beta^t u^{i,t} P_{max}^i) \right] \right\} \quad (5)$$

Hence, for any given λ and β, one can compute the value of $q(\lambda,\beta)$ by solving n_g separate, single-generator dynamic programs of the form

$$\min_{p^{i,\cdot},u^{i,\cdot}} \sum_{t=1}^{n_t} (u^{i,t} F^i(p^{i,t}) - \lambda^t u^{i,t} p^{i,t} - \beta^t u^{i,t} P_{max}^i) \quad (6)$$

These dynamic programs can easily accomodate any $\mathcal{D}$–type constraint such as minimal up or down times and any startup costs. See [4] for the details. Ramp-rate constraints can also be introduced by discretizing the generation range for the unit, although the size of the state space grows considerably.

The separation structure of dual objective suggests using a dual maximization algorithm based on subgradients (which are readily available once the dual objective has been evaluated). The Lagrangian relaxation scheme works fairly well in practice, and through the work of many researchers many advances have been made in the past 20 years. In [5], for example, refinements to the dynamic programming stage are made, and it is shown that the duality gap is expected to be inversely proportional to the number of generators: good news for large scale problems. Other works addressed the inclusion of other kinds of constraints, such as more sophisticated reserve feasibility [7] and line transfer limits by means of a DC flow model of the network [8, 10, 11, 12], and ramping limits [9, 14].

Indeed, the trend to include more and more constraints relies heavily in the structure of the constraint sets. Separability of the dual objective is achieved when constraints that are linear in the optimization variables are relaxed by addding them to the Lagrangian with their respective multiplier. As long as these relaxed constraints are linear or sums of simple functions of single variables, it is always possible to collect terms on a per–generator basis. All system–wide, pointwise in time constraints are natural candidates for relaxation. Unfortunately, nonlinear constraints such as those governing the AC power flow model are not separable through relaxation; neither are line and transformer MVA limits nor voltage limits (because of their reliance on the AC power flow model). Notice, though, that these constraints still fit neatly in the category of $\mathcal{S}$–type constraints. In this paper, we describe an artifact that allows including such constraints into the model and report the progress that has been made in the testing and implementation of the algorithm.

2 Unit commitment with AC nonlinear power flow

Our approach builds on the *variable duplication* technique credited to Guy Cohen in [10] by Batut and Renaud. This is the same technique was used later by

Baldick [11] in his more general formulation of the unit commitment problem. The main difference in our formulation is that reactive power output variables are included, so that better loss management may be performed and generators that are necessary because of their VAr output but not their real power are actually commited. Since one of the sets of variables involved is the solution of a power flow, voltage limits can be included as well.

We start by defining two sets of variables, the *dynamic variables* and the *static* ones:

Dynamic:

- $u^{i,t}$: Commitment status $\{0,1\}$, generator i at time t
- $d_p^{i,t}$: Real power output, generator i at time t
- $d_q^{i,t}$: VAr output, generator i at time t
- U: $(u^{i,t})$, $i = 1 \dots n_g$, $t = 1 \dots n_t$
- D_p: $(d_p^{i,t})$, $i = 1 \dots n_g$, $t = 1 \dots n_t$
- D_q: $(d_q^{i,t})$, $i = 1 \dots n_g$, $t = 1 \dots n_t$
- D: (D_p, D_q)

Static:

- $s_p^{i,t}$: Real power output for generator i at time t
- $s_q^{i,t}$: VAr output for generator i at time t
- S_p: $(s_p^{i,t})$, $i = 1 \dots n_g$, $t = 1 \dots n_t$
- S_q: $(s_q^{i,t})$, $i = 1 \dots n_g$, $t = 1 \dots n_t$
- S: (S_p, S_q)

Then the following optimization problem is defined

$$\min_{D,U,S} \sum_{t=1}^{n_t}\sum_{i=1}^{n_g}[u^{i,t}F^i(d_p^{i,t}) + K^{i,t}(u^{i,\cdot})] \tag{7}$$

subject to:

(I) $\mathcal{D}$–type constraints

$$u^{i,t}P^i_{min} \le u^{i,t}d_p^{i,t} \le u^{i,t}P^i_{max}, \tag{8}$$

$$u^{i,t}Q^i_{min} \le u^{i,t}d_q^{i,t} \le u^{i,t}Q^i_{max}, \tag{9}$$

$$U \text{ satisfies minimal up and down times,} \tag{10}$$

(II) $\mathcal{S}$–type constraints

$$0 \le s_p^{i,t} \le P^i_{max}, \tag{11}$$

$$Q^i_{min} \le s_q^{i,t} \le Q^i_{max}, \tag{12}$$

$$(S_p, S_q) \begin{cases} \text{satisfies network load flow equa-} \\ \text{tions, respecting line MVA \& and} \\ \text{voltage limits} \end{cases} \tag{13}$$

(III) and the following additional constraints

$$R^{l,t} - \sum_{i\in Z_l} u^{i,t}P^i_{max} \le 0, \quad l = 1 \dots n_z, \quad t = 1 \dots n_t \tag{14}$$

$$s_p^{i,t} - u^{i,t}d_p^{i,t} = 0, \quad i = 1 \dots n_g, \quad t = 1 \dots n_t \tag{15}$$

$$s_q^{i,t} - u^{i,t}d_q^{i,t} = 0, \quad i = 1 \dots n_g, \quad t = 1 \dots n_t \tag{16}$$

where $R^{l,t}$ is the minimum combined capacity that is acceptable for the lth zone in the tth period and Z_l is the set of indices of generators in the lth zone.

We will assume that we can enforce both the $\mathcal{D}$ constraints (8–10) and the $\mathcal{S}$ constraints (11–13), so that we only relax the three last constraints (14–16), which leads to the following Lagrangian:

$$\begin{aligned}
\mathcal{L}(U,D,\lambda,\beta) =& \\
& \sum_{t=1}^{n_t}\sum_{i=1}^{n_g}[u^{i,t}F^i(d_p^{i,t}) + K^{i,t}(u^{i,\cdot})] \\
& + \sum_{t=1}^{n_t}\sum_{l=1}^{n_z}\beta^{l,t}(R^{l,t} - \sum_{i\in Z_l}u^{i,t}P^i_{max}) \\
& + \sum_{t=1}^{n_t}\sum_{i=1}^{n_g}\lambda_p^{i,t}(s_p^{i,t} - u^{i,t}d_p^{i,t}) \\
& + \sum_{t=1}^{n_t}\sum_{i=1}^{n_g}\lambda_q^{i,t}(s_q^{i,t} - u^{i,t}d_q^{i,t}) \qquad (17) \\
=& \sum_{i=1}^{n_g}\sum_{t=1}^{n_t}\Big\{u^{i,t}F^i(d_p^{i,t}) + K^{i,t}(u^{i,\cdot}) - \lambda_p^{i,t}u^{i,t}d_p^{i,t} \\
& - \beta^{z(i),t}u^{i,t}P^i_{max} - \lambda_q^{i,t}u^{i,t}d_q^{i,t}\Big\} \\
& + \sum_{t=1}^{n_t}\sum_{i=1}^{n_g}(\lambda_p^{i,t}s_p^{i,t} + \lambda_q^{i,t}s_q^{i,t}) \\
& + \sum_{t=1}^{n_t}\sum_{l=1}^{n_z}\beta^{l,t}R^{l,t} \qquad (18) \\
=& \mathcal{L}_1(U,D,\lambda,\beta) + \mathcal{L}_2(S,\lambda) + \mathcal{L}_3(\beta) \qquad (19)
\end{aligned}$$

where $\lambda = (\lambda_p^{i,t}, \lambda_q^{i,t})$ are multipliers on the relaxed equalities of the two kinds of variables, $\beta^{l,t}$ is the multiplier associated to the lth zone's reserve requirement at the tth period, and $z(i)$ returns the index of the zone to which generator i belongs.

The separation structure of the Lagrangian is obvious upon looking at equations (18) and (19). It makes it possible to write the dual objective as

$$\begin{aligned}
q(\lambda,\beta) &= \min_{U,D,S}\{\mathcal{L}_1(U,D,\lambda,\beta) + \mathcal{L}_2(S,\lambda) + \mathcal{L}_3(\beta)\} \\
&= \min_{U,D}\mathcal{L}_1(U,D,\lambda,\beta) \\
&\quad + \min_{S}\mathcal{L}_2(S,\lambda) \\
&\quad + \mathcal{L}_3(\beta) \qquad (20)
\end{aligned}$$

By looking again at (18) and (20), it can be seen that the first term can be computed by solving n_g dynamic programs again; the second term separates into n_t optimal power flow problems with all generators commited but with special cost curves $\lambda_p^{i,t}s_p^{i,t} + \lambda_q^{i,t}s_q^{i,t}$ for generator i at time t. Notice that $s_q^{i,t}$ also has a price. It is assumed

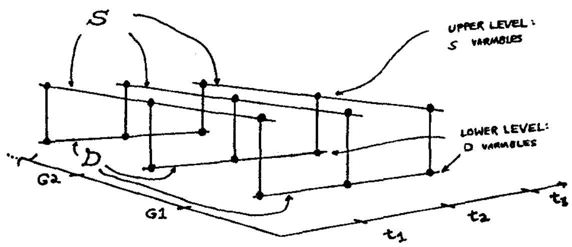

Figure 1: Constraint structure

that the solutions of the dynamic programs meet the $\mathcal{D}$ constraints and that the solutions of the optimal power flows meet the $\mathcal{S}$ constraints.

A graphical description of what we have just achieved can be seen in Fig. 1. The dots represent generator output variables and they are ordered in two dimensions: time period and generator number. The upper layer of variables are the $\{s_p^{i,t}, s_q^{i,t}\}$, while the lower layer are the $\{u^{i,t}d_p^{i,t}, u^{i,t}d_q^{i,t}\}$. The lines represent the connectedness brought by different kinds of constraints; those running parallel to the time axis involve a single generator for all time periods ($\mathcal{S}$–type), while those parallel to the generator index axis involve a single time period and all generators ($\mathcal{D}$–type). Finally, the vertical lines represent the linear equality constraints $s_p^{i,t} - u^{i,t}d_p^{i,t} = 0$, $s_q^{i,t} - u^{i,t}d_q^{i,t} = 0$. When a multiplier is attached to these last constraints in order to relax them, they dissapear from the structure of the dual objective, and all that remains are disjoint (not connected through different kinds of constraints) sets of variables; since costs are separable per generator, the overall separation structure becomes evident.

Before applying a dual maximization procedure to the dual objective as stated, there are some issues that need to be addressed. The first one is that the cost assigned to the $(d_q^{i,t})$ in the dynamic programs, being simply linear, is not strongly convex and that can cause oscillations in the subgradient optimization procedure. Therefore, an augmented Lagrangian technique (where quadratic penalty functions of the relaxed equality constraints are added to the Lagrangian) was used. However, the crossterms now impair the separability, so the Auxiliary Problem Principle described by G. Cohen in [3, 6] was invoked to deal with this. For the details of this treatment, see [15, 12] and the original introduction of this technique to thermal unit commitment problems in [10]. These techniques involve linearization of the nonseparable terms about a previous iteration, modifying them slightly to insure convergence. The resulting Lagrangian is more complicated; after separation, it is given by

$$
\begin{aligned}
&\mathcal{L}(U, D, S, \lambda, \beta, \bar{U}, \bar{D}, \bar{S}) = \\
&\sum_{i=1}^{n_g}\sum_{t=1}^{n_t}\Big\{u^{i,t}F^i(d_p^{i,t}) + K^{i,t}(u^{i,\cdot}) \\
&\quad + \frac{b_p}{2}u^{i,t}(d_p^{i,t})^2 + \frac{b_q}{2}u^{i,t}(d_q^{i,t})^2 \\
&\quad + \left[-\lambda_p^{i,t} - c_p(\bar{s}_p^{i,t} - \bar{u}^{i,t}\bar{d}_p^{i,t}) - b_p\bar{u}^{i,t}\bar{d}_p^{i,t}\right]u^{i,t}d_p^{i,t} \\
&\quad + \left[-\lambda_q^{i,t} - c_q(\bar{s}_q^{i,t} - \bar{u}^{i,t}\bar{d}_q^{i,t}) - b_q\bar{u}^{i,t}\bar{d}_q^{i,t}\right]u^{i,t}d_q^{i,t} \\
&\quad + \left[-\beta^{z(i),t}P_{max}^i\right]u^{i,t}\Big\} \\
&+\sum_{t=1}^{n_t}\sum_{i=1}^{n_g}\Big\{\frac{b_p}{2}(s_p^{i,t})^2 + \frac{b_q}{2}(s_q^{i,t})^2 \\
&\quad + \left[\lambda_p^{i,t} + c_p(\bar{s}_p^{i,t} - \bar{u}^{i,t}\bar{d}_p^{i,t}) - b_p\bar{s}_p^{i,t}\right]s_p^{i,t} \\
&\quad + \left[\lambda_q^{i,t} + c_q(\bar{s}_q^{i,t} - \bar{u}^{i,t}\bar{d}_q^{i,t}) - b_q\bar{s}_q^{i,t}\right]s_q^{i,t}\Big\} \\
&+\sum_{t=1}^{n_t}\sum_{i=1}^{n_g}\Big\{\frac{b_p}{2}\left[(\bar{s}_p^{i,t})^2 + (\bar{u}^{i,t}\bar{d}_p^{i,t})^2\right] \\
&\quad + \frac{b_q}{2}\left[(\bar{s}_q^{i,t})^2 + (\bar{u}^{i,t}\bar{d}_q^{i,t})^2\right]\Big\} \\
&+\sum_{t=1}^{n_t}\sum_{l=1}^{n_z}\beta^{l,t}R^{l,t} \qquad (21) \\
&= \mathcal{L}_1(U, D, \lambda, \beta, \bar{U}, \bar{D}, \bar{S}) + \mathcal{L}_2(S, \lambda, \bar{U}, \bar{D}, \bar{S}) \\
&\quad + \mathcal{L}_3(\beta) \qquad (22)
\end{aligned}
$$

where $\bar{u}^{i,t}$, $\bar{d}_p^{i,t}$, $\bar{d}_q^{i,t}$, $\bar{s}_p^{i,t}$ and $\bar{s}_q^{i,t}$ are the values obtained at the $(k-1)$th iteration. Notice that (22) has the same separation structure of (19). This leads to the following **Algorithm 1:** *AC Augmented Lagrangian relaxation*

1. $k \longleftarrow 0$
2. Initialize $(\lambda_p^{i,t}, \lambda_q^{i,t})$ to the values of the multipliers on the power flow equality constraints at generator buses when running an OPF with all units commited. Initialize $(\bar{U}, \bar{D}, \bar{S})$ to zeros.
3. $(\hat{U}, \hat{D}) \leftarrow \text{argmin}_{\text{feasible } U,D}\, \mathcal{L}_1(U, D, \lambda, \beta, \bar{U}, \bar{D}, \bar{S})$ by solving n_g one-generator dynamic programs.
4. $\hat{S} \longleftarrow \arg\min_{\text{feasible } S} \mathcal{L}_2(S, \lambda, \bar{U}, \bar{D}, \bar{S})$ by solving n_t OPF's in which all generators are committed, their generation range has been expanded to include $P_{min}^i = 0$ and the special cost $\mathcal{L}_2(S, \lambda, \bar{U}, \bar{D}, \bar{S})$ is used. Note: all tasks in steps 3 and 4 can be solved in parallel.
5. If the commitment schedule $\hat{U}$ is not in a database of tested commitments, perform a cheap primal feasibility test. If the results are not encouraging, store the schedule in the database and label it "infeasible", then go to 8.

6. Perform a more serious primal feasibility test by actually attempting to run n_t OPF's with the original P_{min} constraints. If all OPF's are successful, store the commitment in the database, together with the primal cost including startup costs, and the duality gap (the dual cost was available upon solving 3 and 4). Else label the commitment as "infeasible", store it in the database, and go to Step 8.

7. If the duality gap is small enough, stop.

8. Update all multipliers using subgradient techniques, and $\bar{U} \longleftarrow \hat{U}$, $\bar{D} \longleftarrow \hat{D}$, $\bar{S} \longleftarrow \hat{S}$, $k \longleftarrow k+1$

9. Go to Step 3.

3 Implementation details

The complete algorithm was written in $MATLAB^{TM}$ with some FORTRAN subroutines to improve execution speed. The dynamic subproblems can accomodate minimal up or down times, warm start and cold startup costs. The OPF solution code deals with the nonlinear power flow equations, simple box limits on the generators' outputs, voltage limits, line and transformer MVA limits and polynomial cost functions of both the active and reactive generator outputs.

In the early stages of implementation, the static subproblems were solved using Ray Zimmerman and Deqiang Gan's *MATLAB* OPF code [16]. However, since the proposed algorithm needs to compute n_t OPF's every iteration in order to solve the static subproblems, plus extra OPF's in selected iterations when a given commitment is promising, we needed a faster OPF solver. The one outstanding property about the sequence of OPF's to be solved, is that for a given time index, the only data for the OPF that changes is the cost, and only by a relatively small amount dictated by the λ–update. Conceivably, small changes in cost would result in small changes in optimal dispatch, so that it would be advantageous to use the dispatch obtained in the previous iteration as the starting point for the current iteration. In practice, however, we have seen that the dispatch is fairly sensitive to price changes and even the Newton method that we currently use to solve the OPF problems needs several iterations to find the optimum. Furthermore, as the algorithm progresses, some of the costs reflected back to the static subproblems are unusual in the sense that they are unlike any generator's physical economic data. Thus, we have found that the OPF algorithm needs to be especially robust in light of these unusual costs.

Our current OPF implementation is a two stage method. The first stage is a full Newton method with multipliers on the equality constraints and adaptive penalty functions on the inequality constraints. High voltage limits (or, in their absence, the generators' reactive power outputs) are adaptively tightened to push most voltages back into the feasible region; the few remaining outside of the limit are taken to be binding constraints. When equality constraints are met to some small tolerance and voltage limits have been sufficiently tightened to weed out false binding high voltage limits, a guess is taken with respect to which should be the active set and the algorithm switches to the second stage, an active set full Newton method. When problems are encountered, the algorithm tries to find the zeros of the first order optimality condition equations by minimizing a sum of squares of these equations using a specially tailored constrained Levenberg–Marquardt method.

4 Computational results

The algorithm has been tested on modified IEEE 30 and 118 bus systems. For the IEEE 30-bus system [1] with 6 generators, a planning horizon of length 6 was used. For comparison purposes, a version of the Lagrangian relaxation algorithm with DC Flow-based relaxed line limits was also written. The AC-based algorithm correctly identified unit 4 as a must-run because of its ability to provide voltage support for high load periods, even providing some price information on the MVArs that this unit produced by means of the corresponding $\lambda_q^{i,t}$. In contrast, the DC flow-based algorithm failed to commit unit 4 for any period, producing a commitment schedule that was infeasible in light of the AC power flow constraints.

For the 118 bus system with 54 generators, a more complex load variation was used, with three seven–hour days, the two first of them being "weekdays" and the last being "weekend". Each "day" had low nightime loads, average shoulder loads and twin peaks in the morning an afternoon, with a total load variation of -50% to +40% from nominal. Two hundred iterations were run (that is $21 \cdot 200 = 4200$ OPF's). The solutions obtained along the way were of better quality than the corresponding DC power flow Lagrangian relaxation algorithm, partly because in some time periods there were lines operating at their limits and the AC formulation could model these restrictions more accurately. The DC algorithm did not have problems this time finding feasible commitments, partly due to the large number of generators available for dispatch in this network.

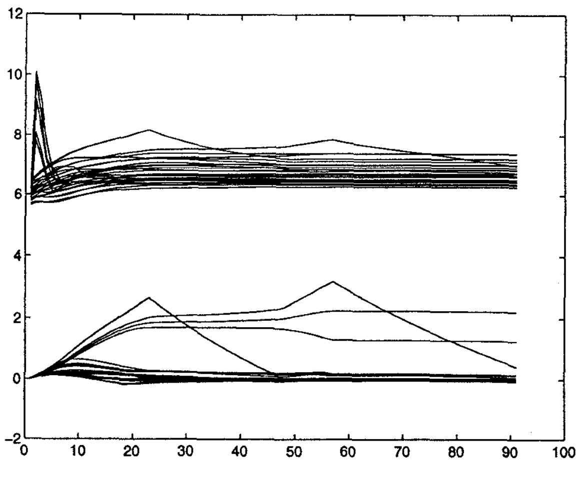

Figure 2: Evolution of multipliers: 30 bus system.

5 Future work

Ramp constraints need to be included in the formulation, and testing of larger scale systems is planned when a parallel processor version of the software is completed.

6 Acknowledgements

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