An Engineering Approach to Monitoring Market Power in Restructured Markets for Electricity

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Abstract

The high average prices and high volatility of prices in many restructured markets for electricity have raised concerns about the abuse of market power by generators. At the same time, information about the true costs of generation, that was readily available under regulation, is no longer disclosed by generators. Hence, it is becoming impractical to use a comparison of actual prices with competitive prices as the basis for identifying the use of market power. In this paper, an engineering procedure is proposed for a given pattern of dispatch to measure the potential for market power for all generators in a network. This procedure is equivalent to a set of factor demand equations in a standard neoclassical model of production. An optimal dispatch, for given sets of offers to sell and constraints on capacity, can be replicated exactly by resolving the dispatch using the optimal nodal prices as offers with no constraints on capacity. Market power exists when the degree of substitutability for power generated at a particular site is low. Withholding capacity and/or raising offers to sell at such a site would be one of the possible ways to exploit market power. Sensitivity of the results for any given pattern of dispatch can give some indication of how effectively prices have been raised by a generator. Under competitive conditions, submitting higher offers to sell at a site will result in a substantial reduction of the capacity dispatched, and the own-price elasticity of demand for generation at the site is very large (negative). As market power increases, the own-price elasticity gets smaller and approaches zero. These effects can be aggregated easily to deal with a specified group of generators, which could represent generators in a load pocket or ownership by a parent company. Use of these procedures is illustrated by evaluating the results of two experiments designed to test market power. These experiments were conducted with undergraduates and utility executives using POWERWEB, which simulates a full AC network with 30 buses, six of which are generators. The objective of the experiments was to determine whether two of the generators could discover that they were in a load pocket and raise their profits by exploiting market power.

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1. Introduction

The prospects for customers in electricity markets are ominous if the events in California since the Summer, 2000 are at all indicative of the future for other regions. Although most customers in the U.S.A. are still paying regulated rates for electricity, customers in San Diego came to the end of the transition period from regulation and were expected to pay the actual market prices for electricity. These market prices were much higher than anticipated, and political intervention was immediate. Price caps in the market were lowered, and customers eventually received rebates. James Hoecker, Chairman of the Federal Energy Regulatory Commission (FERC) is quoted as saying, "Never has this Commission had to address such a dramatic market meltdown as occurred in California's electricity market this summer. Never have residential customers been exposed to economic risk and financial hardship as they were in San Diego" (FERC, 2000). FERC proposed a major modification in the structure of the auction in the California market.

Many of the problems in the Californian market have been recognized by the Market Surveillance Committee (MSC) of the California Independent System Operator (CAISO) (see Wolak, *et al.* [2000]) and in earlier academic papers by Borenstein and Bushnell [1997], Borenstein *et al.* [1999a]) and Borenstein *et al.* (2000). The MSC report includes the following recommendation: "Place greater emphasis . . . on the market-power implications of proposed market rule changes, recognizing that reliability and market-power concerns are inextricably linked." (op. cit. p. 3). When high prices are observed in a market, the possibility that market power is the primary cause is inevitably investigated by regulatory agencies." (For example see Newbery [1995], Wolak and Patrick [1997] and Wolfram [1999] for discussions of the market in the U.K., which was initially manipulated by two dominant suppliers.)

The opportunities for exploiting market power in electricity markets go well beyond the standard concern of having a few firms controlling large shares of production capacity. In fact, Rudbeich *et al.* (1998) and Bunn (1999) suggest that the characteristics of an electricity market make it necessary to have more firms than are needed in a typical market to be competitive. Bunn proposes that firms should be limited to no more than 12% of total production capacity compared to the conventional maximum of 20% (op. cit. p. 20). Other ways to exploit market power include 1) exploiting network characteristics using generators at different locations (Hogan, 1997), 2) using financial contracts when firms are vertically integrated (Pirrong, 2000), 3) using physical bilateral contracts (Joskow and Tirole, 2000), and 4) raising the price of natural gas (New York Times, 2001).

If firms are large enough in any region, there are numerous ways to exploit market power and raise prices above competitive levels.

Many authors have pointed out the limitations of conventional measures of concentration, such as the Hirschman-Herfindahl Index (HHI). (For example, Borenstein *et al.* (1999), p. 80, show how changes in the HHI can be inversely related to market power when large firms withdraw capacity to raise prices, and at the same time, lower the HHI). A better way to measure market power is to compare actual prices with competitive prices.[1] Using this approach, Wolfram (1999) showed that the two large firms in the U.K. market did not exploit the full potential of their market power. For the Californian market, Borenstein *et al.* (2000) estimate that average market prices were about 15 percent above competitive levels from June 1998 to September 1999. (Spot prices in the winter 2000/2001 have been ten times higher.) The problem with analyses of this type is that they require knowing the true costs of production.

Although information on true production costs was readily available under regulation, generators are no longer required to provide data on costs to regulators. Furthermore, treating true costs as private information is consistent with the objectives of deregulation. Competitive markets for other commodities do not require that cost information is made public. If a producer tries to raise prices above competitive levels, the penalty is to lose a lot of market share when a market is competitive. Deregulated markets for electricity should work in exactly the same way, but this will not happen until the underlying causes of market power are understood more fully and then corrected. Hence, it is unfortunate that identifying marketing power is more difficult now under deregulation when the capability to identify market power effectively is needed most.

In summary, the general implications from the literature are 1) the characteristics of electricity markets are atypical, and standard measures of concentration, such as the HHI, are poor indicators of market power, 2) market power can be exploited by suppliers in many different ways that are often dependent on the current operating conditions of the power grid, and 3) there is ample empirical evidence showing that market power has been exploited successfully to raise prices in specific situations in different markets. However, the most effective way of detecting the use of market power, by comparing actual prices to competitive prices, is likely to become less viable in the future because public data on the true costs of production will no longer be available.

[1] The Lerner index is a conventional measure to use, and it measures the percentage increase of actual prices above competitive prices.

The primary objective of this paper is to explore the potential for using an engineering approach to measure the existence of market power in the real-time operations of a power grid. An important feature of this approach is that it requires only information that typically should be available to an Independent System Operation (ISO), such as the actual offers submitted to supply power and the engineering measures needed to determine nodal prices.

Our approach to measuring market power assumes that an ISO solves an Optimal Power Flow (OPF) to determine the least cost pattern of dispatch based on the available offers in a uniform price auction. The OPF is determined subject to physical constraints on the power grid, such as thermal limits on transmission lines, and operating constraints, such as maintaining voltage levels. In the application, a full AC representation of power flows is used so that both real energy and reactive power are reflected in nodal prices, but at this time, the provision of other ancillary services such as reserve generating capacity is not incorporated into the analysis.

Offers to sell power are specified in "blocks" of capacity, implying that a generator submits a specific capacity (e.g., 150 MW) and a price (e.g. $30/MWh) for each block into a uniform price auction. With this structure for offers, a block of capacity may 1) be fully dispatched (nodal price > offer), 2) be partially dispatched (nodal price = offer), or 3) not be dispatched (nodal price < offer). An additional operating constraint is that the first block for each generator must be dispatched fully, or not at all, to represent the minimum operating level for generating power.

The nodal prices corresponding to an optimal pattern of dispatch can be computed for every generator (and every load). These prices incorporate different shadow prices for the operating constraints that are binding, as well as the offers of generators that are selected by the auction (the last accepted offer is used to set the clearing price). The procedures used to determine nodal prices have evolved from the seminal work of Schweppe *et al.*(1988). It is important to note that the nodal prices using a full AC network in our analysis are not the same as the nodal prices using a DC approximation (e.g., the procedure used to allocate transmission shares among generators at a flowgate). The non-linear relationships in an AC network, which are essential for modeling voltage constraints, tend to increase the differences among nodal prices when transmission constraints are binding. Since the potential for exploiting market power also increases when transmission lines are constrained, it may be seriously misleading to use a DC network to measure the physical potential for market power in a network.

Since an OPF determines the optimal pattern of dispatch and the associated nodal prices, it is possible to replicate an optimal solution by replacing offered prices by nodal prices, removing capacity constraints, and resolving the OPF. In this representation, real energy generated at a site can be treated as an input factor into a production process used to meet a specified level of output (i.e., the pattern of loads). Since the objective of an OPF is to minimize the total cost, the derivatives of total cost with respect to the nodal prices for each generator give factor demand equations (by Shepard's Lemma). Elements of the corresponding Hessian matrix of second-order price derivatives represent the complete set of own-price and cross-price effects among generators. These price effects provide the information needed to measure the physical potential for market power implied by the optimal pattern of dispatch determined by an OPF.

The steps needed to derive the Hessian matrix of price effects from an OPF are described in Section 2 of this paper. Since the price effects, and the corresponding measures of market power, apply to the specific patterns of dispatch, load and nodal prices in the OPF, it is desirable to determine how sensitive the results are to changes of the nodal prices. One possible alternative would be to replace the nodal prices with competitive prices. While it is straightforward to compute a new OPF, our underlying assumption is that the information about true costs needed to determine the competitive prices is not available. Two additional OPF solutions are proposed in Section 2 as alternatives for evaluating sensitivity when a load pocket has been identified. These solutions provide some guidance about how effectively the market power in a load pocket has been exploited.

Section 3 describes the results of two experiments on market power using POWERWEB, which simulates an AC network with six generators. This platform has been used extensively to test the performance of different structures for electricity markets. In this application, the objective of the experiments is to determine whether two of the generators can discover that they are in a load pocket and exploit this situation to raise prices. The results show that different groups of undergraduates and a group of utility executives can exploit market power effectively by the end of the experiment.

The experiments in market power provide a convenient basis for testing the engineering measures of market power proposed in Section 2. The results are evaluated in Section 4. They show that the engineering measures are able to identify the two generators in the load pocket relatively easily. Since the true costs are known for this application, the engineering measures can also be determined for the competitive OPF. These results show that the potential for market power can be identified for one competitive OPF but not for the other one, and reasons for this difference are given. It is also possible to determine whether the sensitivity from experimental OPF analyses can identify how effectively market power is exploited. The results about the level of exploitation are more difficult to interpret than the engineering measures for the existence of market power. Developing more informative ways to use the engineering measures is the subject of ongoing research. The conclusions and suggestions for further research are presented in Section 5.

2. An Engineering Measure of Market Power

2.1 The Economics of Production

The duality of production, cost and profit functions has been developed by McFadden and others to form the basis of modern neoclassical production theory (Fuss and McFadden, 1978). Under specific regularity conditions, representing the maximum profit as a function of the output and input prices can be used to derive supply functions for outputs and demand functions for inputs. If outputs are fixed, as they are in our applications, representing the minimum cost as a function of the input prices and the output levels can be used to derive demand functions for inputs. Chapter 7 in Chambers (1988) provides a concise summary of the theory of a multiproduct profit function, and Chapter 2 in Chambers (1988) and Chapter 5 in Varian (1978) discuss the theory of a cost function for inputs.

The cost function for given levels of output, y, n inputs, z, and n input prices, w, can be defined as follows:[2]

$$C(w, y) = \text{Min.} \left(\sum_{i=1}^{n} w_i z_i \right) \textit{w.r.t } z$$

s.t. $F(z) \geq y$

where $z = [z_1, z_2, \ldots, z_n]'$ are the input levels,

$w = [w_1, w_2, \ldots, w_n]'$ are the input prices,

$F(z)$ is a production function.

If the cost function is differentiable, price effects can be derived from the following conditions (Varian op. cit., Sections 1.9-12):

i) $C(w,y)$ is homogeneous of degree one in w,
ii) $C(w,y)>0$ for $w\geq 0$ and $y>0$,
iii) $C(w, y) \geq C(w^*, y)$ for $w \geq w^*$,

[2] For the following discussion, it is assumed implicitly that output levels (loads) are held at fixed levels.

iv) C(w,y) is a concave function of w,

v) $Z(w^*, y) = \nabla_w C(w^*, y) = z^*$ are the optimal levels of inputs that minimize cost (Shepard's Lemma),

vi) Z(w,y) is homogenous of degree zero in w,

vii) If $\{z \geq 0, F(z) \geq y\}$ is a strictly convex set, then Z(w,y) is single-valued,

viii) If Z(w,y) is differentiable, then $M = \nabla_w Z(w^*, y) = \nabla_{ww} C(w^*, y)$ is a symmetric and negative semidefinite Hessian matrix at $\{w^*, y\}$, and $Mw^* = 0$.

The first four conditions define the sufficient conditions for the existence of a cost function. The $n \times n$ Hessian matrix M represents the own-price and cross-price effects for the n inputs (factors). If the elements of M are m_{ij}, then the corresponding price elasticity at $\{w_j^*, z_j^*\}$ is:

$$\in_{ij} = m_{ij} \frac{w_j^*}{z_i^*} = \frac{\partial z_i}{\partial w_j} \frac{w_j^*}{z_i^*}$$

Own Price elasticities, $\in_{ii}$, are non-negative because (viii) above implies M is negative semi-definite. Given the homogeneity condition in (vi) above, $\sum_{j=1}^{n} \in_{ij} = 0$. After converting M to the corresponding matrix of price elasticities, each row of elasticities sums to zero.

For an electrical grid, the pattern of loads on a network corresponds to outputs and the levels of generation correspond to inputs. One of the complications for economists when interpreting standard engineering expressions used in an Optimum Power Flow (OPF) is that P represents the quantity of real energy (MWh) and Q represents the quantity of reactive power (MVar). To avoid this confusion, u is used instead of P for real energy injections in the derivation of M. If nodal prices are computed for the OPF, these prices are the system lambdas. Hence, the Hessian matrix of price effects M has elements $\frac{\partial z_i}{\partial w_j} = \frac{\partial u_i}{\partial \lambda_j}$ in the new notation. However, the derivation of M from an OPF is not straightforward because the OPF is solved subject to a large number of non-linear constraints that define the operating limits of an AC network. The steps used to derive a tractable expression for M are described in Appendix A.

For an individual generator, i, the marginal change of profit with respect to the nodal price is:

$$d(\lambda_i u_i - C_i) = (u_i + \lambda_i m_{ii} - C_i' m_{ii}) d\lambda_i \quad (1)$$

where λ_i is the nodal price

u_i is the real energy dispatched

$C_i = C_i(u_i)$ is the total cost of production

C_i' is the corresponding short-run marginal cost

$m_{ii} = \frac{\partial u_i}{\partial \lambda_i} \le 0$ is the own price effect from the Hessian matrix M.

Expression (1) can be rewritten in terms of the own-price elasticity as follows:

$$d\,Profit_i = Revenue_i \left(1 + \left(1 - \frac{C_i'}{\lambda_i}\right) \in_{ii}\right)\left(\frac{d\lambda_i}{\lambda_i}\right) \qquad (2)$$

Consequently, profit will increase when the nodal price increases if:

$$\left(1 + \left(1 - \frac{C_i'}{\lambda_i}\right) \in_{ii}\right) > 0 \qquad (3)$$

In most situations, $\lambda_i > C_i' > 0$, because λ_i is determined by the offers and not the true marginal costs, and $0 < \left(1 - \frac{C_i'}{\lambda_i}\right) < 1$. Since $\in_{ii} \le 0$, the condition in (3) is equivalent to $\in_{ii} > -1/(1 - C_i'/\lambda_i)$. If all generators submit honest offers equal to the marginal costs, then at least one generator will have $\lambda_i = C_i'$, implying that the condition is $\in_{ii} > -\infty$, and thus, not informative. A more stringent criterion is to consider whether $Revenue_i$ increases when λ_i increases. Using this more restrictive requirement, the potential for market power exists if $-1 < \in_{ii} \le 0$ for any value of $C_i' \ge 0$. This provides a sufficient condition for generator i that is not dependent on knowing the true costs and it corresponds to assuming the marginal cost is zero.

The proposed condition for an individual generator implies that additional profits can be made if the net-demand for power from that generator is inelastic. This is a standard result in microeconomics, and it corresponds to an economic version of a must-run unit. However, this condition is too stringent as a test for market power because a small group of generators may also possess market power. In other words, even if competition exists within the group, the group itself may be small enough in number to exploit market power. In practice, such groups are likely to be in sub-regions that have limited transmission capacity for importing power or represent generators owned by one company.

Consider the following situation in which the first r<<n generators in a network may possess market power. For the ith generator in the group, the marginal change in profits with respect to proportional increases of all nodal prices in the group is:

$$
\begin{aligned}
d(u_i\lambda_i - C_i) &= \left[u_i d\lambda_i + \sum_{j=1}^{r}(\lambda_i m_{ij} - C_i' m_{ij})d\lambda_j\right] \\
&= \left[u_i\lambda_i\left(\frac{d\lambda}{\lambda}\right) + (\lambda_i - C_i')\sum_{j=1}^{r}\lambda_j m_{ij}\left(\frac{d\lambda}{\lambda}\right)\right] \qquad (4) \\
&= \text{Revenue}_i\left[1 + \left(1 - \frac{C_i'}{\lambda_i}\right)\sum_{j=1}^{r}\in_{ij}\right]\left(\frac{d\lambda}{\lambda}\right)
\end{aligned}
$$

The same argument below (3) can be used to make (4) independent of information about costs. The resulting condition can be stated:

Revenue (and profit) for generator i in the group will increase if:

$$\sum_{j=1}^{r}\in_{ij} > -1 \qquad (5)$$

The potential for market power exists for a group of r generators if

$$\sum_{i=1}^{r}\text{Revenue}_i\left[1 + \sum_{j=1}^{r}\in_{ij}\right] > 0 \qquad (6)$$

It is clear that this condition (6) holds if all of the generators in the summation in (6) meet condition (5), but this is not a necessary condition, particularly if one company owns more than one of the generators in the group.

The correct size of the group, r, is difficult to define precisely because expression (6) is likely to hold if r is large enough, and it holds with certainty for $r = n$. Hence, r must be relatively small and represent situations in which tacit collusion among generators is likely to be successful. For example, $r = 5$ is a reasonable value to start with for a network, and this value is still smaller than the minimum number of 8 generators proposed by Bunn (1999) for a competitive market. In practice, an ISO would know enough about the characteristics of a network to identify likely load pockets in advance, but the potential for exploiting market power by owning generators at different locations on a network is also important (Hogan (1997)). Observing repeated high nodal prices, compared to the average prices paid, is an obvious criterion for identifying generators in a load packet. Knowledge of ownership patterns is the other way to determine potentially interesting groups of generators. The effectiveness of the proposed conditions (5) and (6) is demonstrated in Section 4 using the results of the two experiments. These experiments, which are described in Section 3, focus on the potential for market power in a load pocket.

3. Testing Market Power Using POWERWEB

3.1 The Design of Experiment I

Two sets of experiments were conducted to examine whether two generators inside a load pocket could identify and exploit the reduced intensity of competition caused by a transmission constraint. Each experiment employed a six generator, 30-node alternating current (AC) network simulated by POWERWEB (a description is provided in the Appendix B). In each experiment, the network was divided into two regions (A and B) joined by transmission links with finite thermal capacities. Each region had a separate demand that could be satisfied by any of the six generators operating subject to the transmission constraints. Figure 3-1 shows a schematic representation of the network for the first set of experiments.

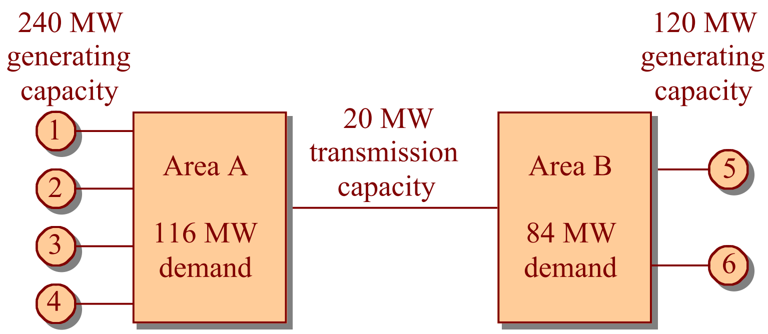

Figure 3-1: Transmission Network Block Diagram

Area B is the load pocket. Served by only two generators with only limited competition from the other four generators, prices in this region are expected to be higher than competitive levels. In addition to the limit on imports into Area B, the two generators inside the load pocket are imperfect substitutes. There are actually two transmission lines into the load pocket. If either of the generators inside the load pocket generates less, the net demand at the end of one transmission line is increased. This causes increases in the flow of energy towards the node where the demand has increased, including flows from outside of the load pocket. In order to prevent the line from being overloaded when it is already at full capacity, some capacity outside the load pocket must be backed down, representing an additional cost to the system. Generators in the load pocket can raise their offers until the marginal cost of their energy equals the marginal cost of backing down generation outside the load pocket. The transmission constraint, therefore, has both a direct effect that limits flows of real energy into the region and an indirect effect caused by voltage support that affects nodal prices for all generators.

Table 3-1 shows the cost parameters and block sizes for the six generators in the first experiment. Each generator was divided into three blocks of capacity with different marginal costs. Generators 1 through 4 were located outside the load pocket in Area A. Generators 5 and 6 were located inside the load pocket in Area B. Note that in this experiment, the six generators are identical in cost and size. However, the configuration of this network implies that generator 6 is in a more favorable location than generator 5. Generator 6 will be backed down at a slower rate than generator 5 in response to making higher offers.

Table 3-1: Generator Costs and Capacities for Experiment I

Generator	*Block 1*		*Block 2*		*Block 3*	
	MW	**$/MW**	**MW**	**$/MW**	**MW**	**$/MW**
1	12	20	24	40	24	50
2	12	20	24	40	24	50
3	12	20	24	40	24	50
4	12	20	24	40	24	50
5 – Inside Load Pocket	12	20	24	40	24	50
6 – Inside Load Pocket	12	20	24	40	24	50

Given the costs in Table 3-1, the competitive market solution corresponds to generators submitting offers equal to the true marginal costs. The competitive solution is shown in Table 3-2. Using the Last Accepted Offer (LAO) to determine the market clearing price, the second blocks of generators 1 and 3, which are partially dispatched, set the market price at $40/MWh. The higher nodal prices paid to the other generators reflect network constraints and losses. However, the transmission constraints from region A to region B are not binding because only 12 MW of real energy is transmitted on lines that have a maximum capacity of 20 MW.

Table 3-2: The Competitive Market Solution for Experiment I

Generator	*MW Dispatched*	*Nodal Prices $/MWh**	
		LAO	FRO
1	23.96	40.00	48.40
2	36.00	40.08	48.48
3	34.90	40.00	48.40
4	36.00	40.13	48.53
5 – Inside Load Pocket	36.00	41.10	49.50
6 – Inside Load Pocket	36.00	41.60	50.00

* LAO Last Accepted Offer
FRO First Accepted Offer

Since the aggregate marginal cost curve for all generators is a step function, generators can raise the competitive price above the LAO, without altering the efficient pattern of dispatch, until the next most expensive block is reached. This latter situation corresponds to setting the market price to the First Rejected Offer (FRO) of $50/MWh for generator 6. The difference between the LAO and FRO prices represents the potential range of competitive prices. In the experiments, the expectation is that prices outside the load pocket will be in this range, and that prices in the load pocket will be higher than the FRO. Note that the difference among LAO prices and among FRO prices are small because flows on the network are relatively unconstrained.

Total system demand is 200 MW and it is completely inelastic[3]. The load pocket has a demand of 84 MW and each of the generators in the load pocket has a capacity of 60 MW. If the transmission line is constrained, which it will be if the price inside the load pocket is above the price outside the load pocket, then only 20 MW of capacity can be imported into the load pocket. This leaves a residual of 64 MW of load that must be divided between 120 MW of generating capacity. Since the required generation of 64 MW in the load pocket is greater than the 60MW capacity of a single generator, both generators in the load pocket are essential. If one generator raises its offer to the maximum allowed price (the reservation price of $80/MWh) and the other generator submits offers below the reservation price, then the latter generator will be fully dispatched at 60 MW. 4 MW will be supplied by the generator submitting offers at the reservation price, and this will set the marginal price in the load pocket. Additional capacity cannot be imported from outside the load pocket even if the offers there are much lower. The actual situation is slightly more complicated because generators have a minimum capacity requirement corresponding to the first block of 12MW. Since both generators are essential in the load pocket, each generator must operate at least the first block of capacity even if the offer is at the reservation price. von der Fehr and Harbord (1993) point out that one of the generators in this type of situation can increase its profits by submitting lower offers. By selling more capacity at a high price (set by the other generator) profits are increased. This position can be maintained by offering capacity at a price low enough to ensure that the other generator's payoff is reduced if it tries to undercut the low offers.

[3] At present most deregulated markets model demand to be perfectly inelastic. Only a small percentage of the load is price sensitive and so the assumption of perfect inelasticity in the experiments is not unreasonable.

3.2 The Results for Experiment I

This experiment was conducted for five different groups of undergraduates and one group of utility executives. For each session, the same auction was repeated 75 times. Individual participants knew only 1) the costs and capacities of their own generators, 2) the total system demand, and 3) that there were five other competitors in the market. They knew nothing about the network or the potential load pocket. Figure 3-2 shows a comparison of the average prices inside and outside the load pocket for all of the undergraduate sessions. Prices in the load pocket, by the end of the experiment, were much higher than those outside of the load pocket. The line at $50/MWh represents the maximum competitive price (FRO) if all generators had submitted offers at the true marginal costs. Since the corresponding minimum competitive price (LAO) is $40/MW, prices between $40/MWh and $50/MWh can be considered competitive.

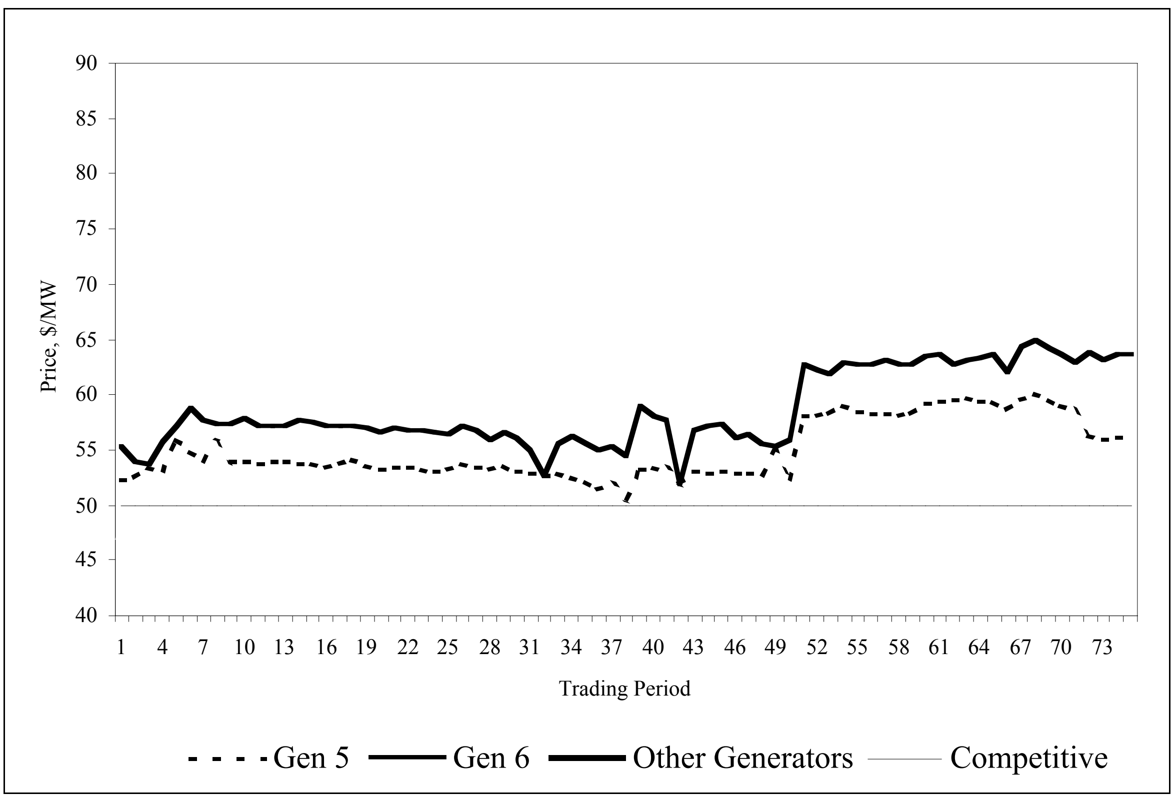

Figure 3-2: Average Prices in Undergraduate Sessions: Experiment I

The average price outside the load pocket is rarely above the competitive range. The fact that prices are close to the upper boundary of competitive prices would suggest that with only four participants the market is not likely to be perfectly competitive. This is supported by previous experimental examinations of LAO multiple unit auctions by Bernard *et al.* (1998) who show that at least six participants with a fixed inelastic load are needed to be efficient. Inside the load pocket, prices are on average slightly higher than the upper boundary of competitive prices until the last twenty rounds of the experiment. During the last twenty rounds average prices in the load pocket jump, with generator 6 (as expected) being the price leader. The lower price for generator 5 reflects locational differences in nodal prices due to voltage support. This price difference is much larger than the differences shown in Table 3-2 because of the effects of transmission constraints on voltage.

Figure 3.3 shows the same comparison of prices from the experiment with utility executives. Note that the same general result is achieved with prices outside the load pocket approximately competitive while prices insides the load pocket are substantially higher (with generator 6 leading in price). The main differences between the executives and the undergraduates are the speed with which the executives identified the degree of market power they possessed, the higher prices achieved inside the load pocket, and the use of signaling by generator 6. Dropping the offers from the reservation price to marginal cost in some periods is an attempt to get generator 5 to raise its offers. In duopoly experiments, Bernard *et al.* (1998) found that the two identical participants generally reached some tacit agreement to share the responsibility for raising the market price. However, in this case, there are locational differences between generators 5 and 6. The behavior of generator 5 is more like the strategy proposed by von der Fehr and Harbord (1993). A more complete analysis of these results is given in Ede *et al.* (2000).

There were noticeable differences in how well individual sessions with the undergraduates exploited market power. Using the average price paid to generators over the last 10 periods of each session as a criterion, the utility executives were paid prices of $63/MWh and $76/MWh for generators 5 and 6, respectively. One of the undergraduate sessions received even higher prices. In contrast, two of these sessions received prices that were not significantly higher than the competitive FRO of $50/MWh. Finding the load pocket required some bold exploratory behavior. For example, generator 5 exhibits such behavior in the early periods in Figure 3.3. Generator 6 realizes that something is going on because sometimes high prices are paid, but these high prices for generators 5 and 6 do not have much effect on the prices paid to generators 1-4. If generators 1-4 follow the same behavior as generator 5, they simply lose market share and may be shut down. Learning in actual

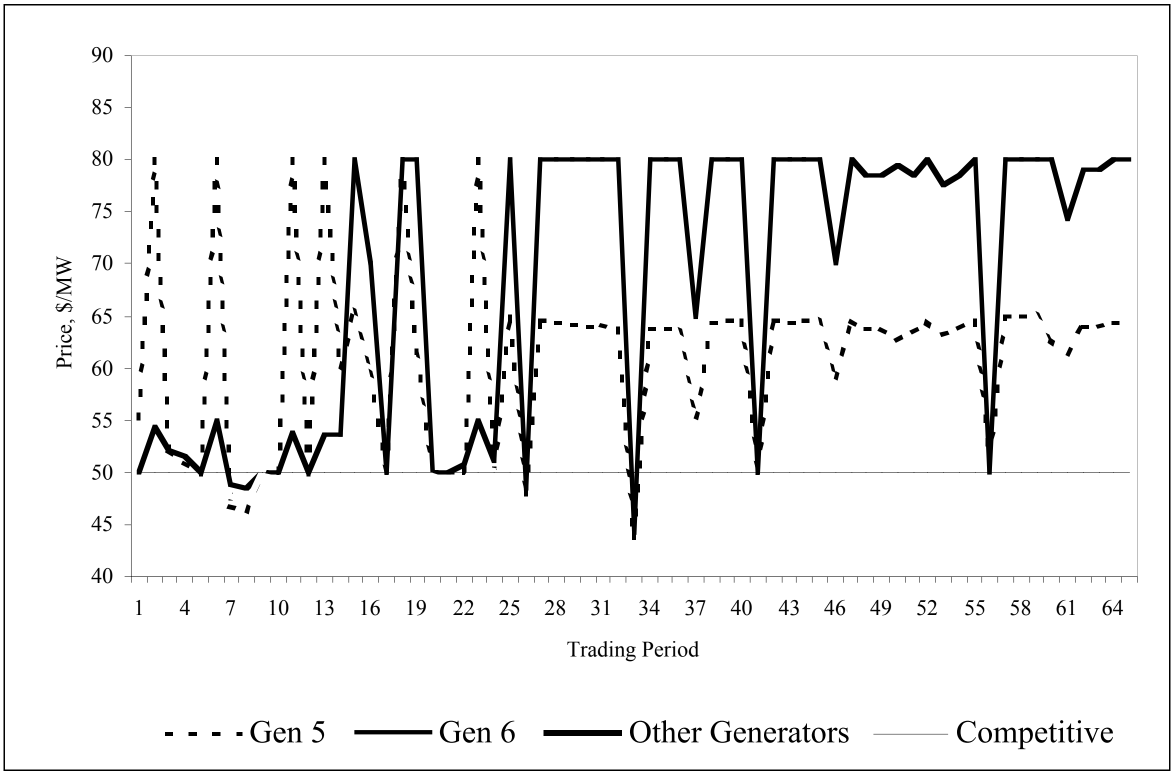

Figure 3-3: Average Prices in Utility Executive Session: Experiment I

markets is accelerated by observing prices in other markets. Once traders see what is possible (e.g., getting price spikes), it does not take long for them to figure out how to duplicate the price behavior if their own market is vulnerable to exploitation (i.e., market power exists).

3.3 The Design of Experiment II

Once the load pocket has been discovered in Experiment I, it is relatively easy for generators in the load pocket to exploit market power. Since both generators 5 and 6 eventually find out that their services are essential to meet demand, they can increase their profits by raising their offers. The only competition between them will be to determine how much of their capacity is dispatched to meet the net-load of 64 MW. A second set of experiments was conducted to examine a case in which cost differences result in a binding transmission constraint which creates a load pocket for generators 5 and 6. The market was again partitioned into two areas (A and B) in Figure 3-1 with area B being the load pocket. Demand throughout the system is completely inelastic, but the demand inside the load pocket was reduced from 84MW to 49MW. If one generator was completely shut down, demand could be entirely satisfied by the other generator. This would ensure that neither generator inside the load pocket was essential to the operation of the system. Exploiting the load pocket in Experiment II requires tacit collusion between generators 5 and 6.

The transmission line was constrained because the marginal costs for the two generators inside the load pocket were higher than the marginal costs of the generators outside the load pocket. With marginal cost offers, energy would flow to the maximum extent into the load pocket from outside, and power flow along the transmission line would be at its maximum of 20MW. This situation is typical for many urban load centers that import inexpensive power from remote baseload generators. Table 3-3 shows the new cost and capacity parameters of each generator in the experiment. The changes from Experiment I are bold.

Table 3-3: Generator Costs and Capacities for Experiment II

Generator	*Block 1*		*Block 2*		*Block 3*	
	MW	**$/MW**	**MW**	**$/MW**	**MW**	**$/MW**
1	12	20	24	40	24	50
2	12	20	24	40	24	50
3	12	20	24	40	24	50
4	12	20	24	40	24	50
5 – Inside Load Pocket	**12**	**45**	**24**	**55**	**24**	**60**
6 – Inside Load Pocket	**12**	**45**	**24**	**55**	**24**	**60**

The competitive solution for Experiment II is summarized in Table 3-4. Each of the generators inside the load pocket sell at close to the minimum generating limits, and the transmission lines are used to the maximum of 20 MW. There are, in effect, two separate markets, and different blocks set the prices inside and outside the load pocket. The ranges of competitive LAO and FRO prices are $40/MWh to $50/MWh outside the load pocket, and $54.27MWh to $55.73/MWh inside the load pocket.

Table 3-4: The Competitive Market Solution for Experiment II

Generator	*MW Dispatched*	*Nodal Prices $/MWh*	
		LAO	FRO
1	31.72	40.00	49.85
2	36.00	40.15	50.00
3	34.03	40.00	49.85
4	36.00	40.14	49.99
5 – Inside Load Pocket	17.57	55.00	55.73
6 – Inside Load Pocket	12.00	54.27	55.00

* LAO Last Accepted Offer
FRO First Accepted Offer

Having two markets in the competitive solution changes the nature of the indirect market power associated with the transmission constraint. In the previous experiment, changes in the dispatch levels of generators 5 and 6 in the competitive solution affected the flow of energy throughout the system, and altered the dispatch of some generators outside the load pocket. In this experiment, the effects of changes inside the load pocket on generators outside the load pocket are small. Taking into account constraints on the transmission network, if either generator 5 or 6 submits offers for both blocks 1 and 2 above $55/MWh, the market can adopt the characteristics of Bertrand price competition, with the other generator possessing sufficient capacity to undercut the offer and be the only generator dispatched in the load pocket. In Experiment I, this was the way to discover that block 1 was essential. Consequently, raising prices in the load pocket in Experiment II requires tacit collusion between generators 5 and 6.

3.3.4 The Results of Experiment II

This experiment was conducted for six different groups of undergraduates, and the auction was repeated 75 times in each session. Once again the students were not told about the network or the load pocket. Figure 3-4 compares the average prices outside the load pocket across all of the undergraduate sessions against the average prices received by generators 5 and 6, inside the load pocket. All four generators outside the load pocket sell most of their second block of capacity. Competition for load would be with the third blocks of other generators which are priced at $50/MWh, and the range for competitive prices is $40-$50/MWh. It is evident in Figure 3-4 that the average price outside the load pocket converged on the upper boundary.

Inside the load pocket, only a small proportion of one of the generator's second block is needed to meet the net load of (49 - 20) = 29 MW, leaving a large balance of unused capacity (2 x 24 – 5 = 43 MW) priced at $55/MWh. This would be, therefore, the price at which competition for load occurred. Given that it takes a number of rounds for the subjects in the experiment to understand the workings of the auction and the extent of market power, it is unlikely that the experiment was long enough for the generators to exploit market power fully. The results in Figure 3-4 show that both of the generators inside the load pocket were able to gradually raise their prices throughout the experiment. It is clear that prices ended up higher than competitive levels, and it is likely that they would have gone even higher if the experiment had continued. Additional analyses of these results are presented in Ede *et al*. (2000).

There were differences among the six sessions in the final levels of prices reached in the load pocket. Using the average price paid for the last 10 periods as a criterion, the prices for generator 5 ranged from \$61/MWh to \$72/MWh, and from \$55/MWh to \$71/MWh for generator 6. (In Experiment II, generator 5 had the most favorable location in the load pocket for exploiting market power, and the differences in prices between generators 5 and 6 reflect the spatial effects of nodal pricing using an AC network). The main difference from Experiment I was that discovering market power in Experiment II did not require bold behavior because the load pocket already existed in the competitive solution. Since neither generator 5 nor 6 are essential, as they were in Experiment I, the exploitation of market power in Experiment II evolved gradually throughout the experiment.

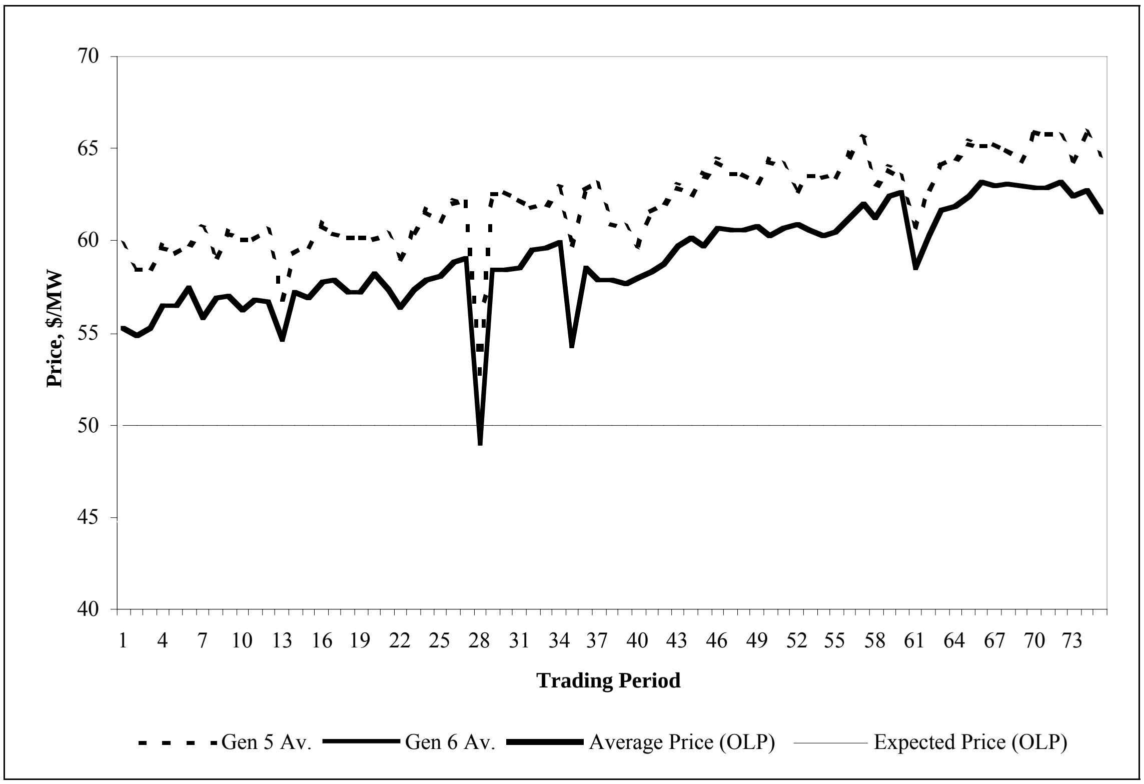

Figure 3-4: Average Prices in Undergraduate Sessions: Experiment II

4. Applications of the Engineering Measures

4.1 The General Approach

The two experiments discussed in the previous section showed that two generators in a load pocket could exploit market power effectively and raise prices above competitive levels. The four remaining generators were able to raise prices to the high end of the competitive range ($50/MWh) but not higher. In the first experiment, there were no major binding constraints on transmission in the competitive solution, and as a result, there is no load pocket in this situation. The load pocket had to be created by having generators 5 or 6 submit high offers that cause imports to increase until transmission constraints are binding. Once this has been accomplished, however, both generators in the load pocket were essential to meet load. For these two generators, the price for the first block of capacity could always be raised to the maximum allowed in the auction by submitting all offers at the reservation price ($80/MWh).

In the second experiment, the competitive solution exhibited a load pocket because the true costs of generation were higher for the two generators in the pocket than they were for the other four generators. Economic efficiency required that power should be imported from outside the load pocket to the maximum amount possible. Consequently, all transmission lines into the load pocket were run at full capacity in the competitive solution. Neither generator in the load pocket was essential, but they represented a duopoly for supplying the remaining power needed to meet load. Consequently, they were able to raise prices through tacit collusion (e.g. Nash bargaining).

The two experiments represent completely different situations in the competitive cases, and it should be possible to identify market power in Experiment II but not in Experiment I. Using the procedures described in Section 2 and Appendix A, the corresponding matrices of price elasticities are computed for the two competitive solutions to illustrate the differences. For each experiment, an additional OPF was run to represent the session that was most successful in exploiting market power (using the highest prices received in the last 10 trading periods as the criterion). The corresponding matrices of price elasticities are computed to compare with the competitive cases. For this comparison, the competitive solution can be viewed as a case in which generators were unable to exploit market power.

Using the matrices of price elasticities, it is relatively easy to identify the cases in which the physical potential for market power exists. However, it is more difficult to determine how effectively market power has been exploited. Having identified the two generators with market power, the first sensitivity run is to solve an OPF with all offers for these two generators set at the reservation price. This result shows whether or not the two generators are essential (i.e. do not lose market share), and if they are essential, one can conclude that the market power is not a local phenomenon for the specific conditions in the original OPF. It would also be possible to submit offers at the reservation level for just one generator in the load pocket at a time, and to compute the OPF to determine whether that individual generator is essential.

Consider the following situation in which 1) a load pocket has been identified, and 2) it is more than just a local phenomenon at the observed OPF. It is possible to compare the observed nodal prices with the corresponding prices when offers in the load pocket are at the maximum. If the two sets of nodal prices are close together, one could argue that market power has been exploited effectively. One possible measure of exploitation would be the percentage of the maximum revenue (i.e., with offers at the reservation price) actually paid to the generators in the load pocket. This measure would, however, be very sensitive to the level of the reservation price (price cap) in a market. A better procedure might be to adopt a specified "high" price as a standard for the proposed measure of exploitation.

Clearly, the ideal measure of market power is to compare the observed nodal prices with the competitive prices, but this ideal is unattainable under deregulation. An alternative is to measure the opportunity cost for power imported into the load pocket. For the second sensitivity run offers for the two generators in the load pocket are set at zero, but the capacities offered are constrained to the optimum levels in the original OPF plus epsilon. The small increase of generation in the load pocket will relax the transmission constraints, and as a result, the nodal prices for the two generators in the load pocket will correspond to the cost of imported power. Even if these nodal prices are considered to be crude substitutes for the true competitive prices, they would still be useful for evaluating the potential benefits of expanding transmission capacity into the load pocket. For our purposes, when the nodal prices for the case with zero offers are much lower than the corresponding prices in the observed OPF, the cause may be market power.

Combining the results for the observed OPF and the two sensitivity runs, with the high offer (\$80/MWh) and the low offer (\$0/MWh), respectively, it is possible to calculate the following measure of Relative Market Power (RMP):

$$\mathbf{RMP} = \mathbf{100}\left[\frac{\textbf{Observed price} - \textbf{Low price}}{\textbf{High price} - \textbf{Low price}}\right].$$

High values of RMP close to the maximum of 100 indicate that market power has been exploited successfully. Although the RMP works quite well for our examples, it is still not an ideal measure. Developing better measures of the exploitation of market power is one of the ongoing objectives of our current research. It should be noted, however, that the main limitation of the RMP is the inability to discover the true costs. This is a deficiency on the supply side. From the perspective of customers, the prices paid are more important than measuring profits. Hence, the RMP, or, as an alternative, the ratio [observed price/low price], provides a reasonably good measure of how well the power system is working for customers in a load pocket.

4.2 Identifying the Existence of Market Power

For both experiments, the six generators can be grouped into three regions based on the layout of the network shown in Appendix B. (The 2x2 blocks of price elasticities for generators in the same region are bold in Tables 4.1 and 4.2). Hence, it will be useful to identify the differences within regions from the differences across regions. In general, one would expect the degree of substitutability between two generators to be inversely related to how far apart they are on the network.

The elasticities for the Competitive solution in Experiment I (Table 4.1) represent a situation with no major binding constraints on transmission (i.e. no load pockets). The own-price elasticities are all large (>12 in absolute terms), showing that individual generators are highly competitive. Generators 1-4 are relatively close substitutes ((i.e. have large positive cross-price elasticities), and so are generators 3-6. In contrast, generators 1 and 2 have small complementary relationships with generators 5 and 6.

Table 4.1 Matrices of Price Elasticities for Generators: Experiment I

Competitive Solution						
	1	2	3	4	**5**	**6**
1	**-141.1**	**131.6**	5.3	4.7	-.1	-.3
2	**87.4**	**-113.3**	15.2	11.6	-.1	-.8
3	3.7	15.7	**-29.1**	**2.4**	4.8	2.5
4	3.1	11.5	**2.3**	**-21.0**	2.6	1.5
5	-.1	-.1	4.6	2.5	**-16.6**	**9.7**
6	-.2	-.7	2.3	1.4	**9.6**	**-12.4**
Dispatch (MW)	24.0	36.0	34.9	36.0	36.0	36.0
Price ($/MWh)	40.0	40.1	40.0	40.1	41.1	41.6
Actual Experiment						
1	**-167.1**	**162.4**	1.8	3.1	5.5	-5.7
2	**77.3**	**-88.5**	5.4	6.0	6.7	-6.9
3	.9	5.6	**-14.8**	**8.0**	-6.4	6.7
4	1.5	6.1	**7.8**	**-15.6**	-3.4	3.6
5	2.5	6.3	-5.8	-3.2	**-6.9**	**7.1**
6	-1.9	-4.8	4.4	2.4	**5.2**	**-5.3**
Dispatch (MW)	19.6	41.0	37.9	39.2	28.8	36.8
Price ($/MWh)	49.6	49.7	51.5	51.2	75.0	79.5

The elasticities for the Competitive Solution in Experiment II represent a case in which generators 5 and 6 are in a load pocket. However, all of the own-price elasticities are still very large in absolute terms (>10) and individual generators do not appear to have market power. The pattern of cross-price elasticities is more complicated than it was in Experiment 1. The most notable features are the relatively large negative and positive cross-price elasticities that occur between generators 5 and 6 in the load pocket and the other four generators. The load pocket is not cut off from the rest of the network (i.e. with cross-price elasticities close to zero) as one might expect. The reason is that the transmission constraints make it harder for the network to maintain voltage constraints. Changes of generation within the load pocket can still have important effects outside the load pocket, even though the amount of power imported into the load pocket stays the same.

Table 4.2 Matrices of Price Elasticities for Generators: Experiment II

	1	2	3	4	5	6
Competitive Solution						
1	**-105.0**	**101.9**	1.0	2.0	1.6	-1.5
2	**89.5**	**-103.7**	6.8	7.3	8.7	-8.6
3	.9	7.2	**-17.0**	**9.1**	-6.9	6.7
4	1.8	7.3	**8.5**	**-17.6**	-3.5	3.5
5	2.2	13.1	-9.7	-5.3	**-10.1**	**9.8**
6	-3.0	-19.1	14.1	7.7	**14.6**	**-16.0**
Dispatch (MW)	31.7	36.0	34.0	36.0	17.6	12.0
Price ($/MWh)	40.0	40.1	40.0	40.1	55.0	54.3
Actual Experiment						
1	**-87.9**	**85.4**	.4	2.0	1.5	-4.4
2	**92.3**	**-106.5**	6.8	7.2	9.1	-8.9
3	.5	8.0	**-19.9**	**11.7**	-7.5	7.2
4	2.2	7.2	**10.0**	**-19.3**	-4.2	4.1
5	2.6	14.4	-10.1	-6.6	**-12.5**	**12.2**
6	-2.6	-14.8	10.3	6.8	**12.9**	**-12.6**
Dispatch (MW)	37.9	34.9	30.1	34.9	14.9	14.6
Price ($/MWh)	48.5	48.7	48.5	48.6	72.0	70.0

The key feature of the load pocket for the Competitive Solution in Experiment II is that the own-price and cross-price elasticities for generators 5 and 6 are roughly the same size (but with opposite signs). The results in Table 4.3 summarize the net elasticity for each generator with the other generator in the same region

Table 4.3 Net Elasticities for Each Generator

	Experiment I		Experiment II	
Generators	Competitive	Actual	Competitive	Actual
1 with 2	-9.5	-4.7	-3.1	-2.5
2 with 1	-25.9	-11.2	-14.2	-14.2
3 with 4	-26.7	-6.8	-7.9	-8.2
4 with 3	-18.7	-7.8	-9.1	-9.3
5 with 6	-6.9	+.2	-.3	-.3
6 with 5	-2.8	-.1	-1.4	+.3

(e.g., for generators 1 with 2 for the Competitive Solution in Experiment I, the net elasticity is (–141.1 + 131.6 = -9.5)). For the unconstrained competitive case (Competitive Solution in Experiment I), all net elasticities are <-2.8, and fall outside the range proposed in condition (5) (see Section 2) to identify the existence of market power (i.e. net elasticity >-1). For the other three cases, all of the net elasticities for generators 1-4 fall well outside the range. In contrast, five of the six net elasticities for generators 5 and 6 are within the range, and two of them are actually positive. The one exception for generators 6 with 5 for the Competitive Solution in Experiment II (-1.4) is still close to –1, and the combined criterion for generators 5 and 6, shown in condition (6), implies that market power does exist (55.0 x 17.6 (1 – 10.1 + 9.8) + 54.3 x 12.0 (1 - 16.0 + 14.6) = 708.06 > 0). For the two Actual Experiments, the results for both generators 5 and 6 give strong support for the existence of market power because the net elasticities are well above –1.

The overall conclusion is that the measures proposed in Section 2 identify the existence of market power correctly. The results in Table 4.3 show that market power exists for generators 5 and 6 in all cases except the Competitive Solution in Experiment 1. In contrast, generators 1 and 2 do not have market power in any of the four cases, and similarly, generators 3 and 4 do not have market power. However, the combined group of generators 1-4 must have market power when generators 5 and 6 have market power because the rows of elasticities sum to zero (see Section 2). The implication is that the two generators inside the load pocket will find it much easier to raises prices than the four generators outside the load pocket. However, in a complicated network facing different types of uncertainty, there is no guarantee that four generators will be competitive even though they did behave competitively in our experiments (one possible reason is that load was not stochastic in the experiments).

4.3 Measuring the Exploitation of Market Power

The results in Section 4.2 show that generators 5 and 6 have market power in the Actual Experiments. These are the cases that an ISO would observe. Hence, the next question is whether or not generators 5 and 6 are using their market power effectively to raise prices. Seeing prices for generators 5 and 6 substantially higher than the prices paid to other generators may raise suspicions, but, this situation is neither sufficient nor necessary for exploiting market power.

The two additional sensitivity runs proposed in Section 4.1 identify a high offer case (offers for generators 5 and 6 set to the reservation price of $80/MWh) and a low offer case (offers for generators

5 and 6 set to \$0/MWh). The nodal prices and optimal levels of dispatch for generators 5 and 6 in the two different cases are shown in Table 4.4 for Experiment I and Table 4.5 for Experiment II. For Experiment I, the total dispatch for generators 5 and 6 is lower in the Actual Experiment (65.6MW) than it is for the Competitive Solution (72.0MW), because this is how the load pocket was created. The total dispatch in the High Offer case is still (65.5MW), implying that market power is persistent and not just a local phenomenon (the corresponding matrix of elasticities for the High Offer case also confirm these conclusions).

Table 4.4 Sensitivity Analysis for Generators 5 and 6: Experiment I

	Generator	Competitive	Actual	High Offer[1]	Low Offer[2]	RMP
Price	5	41.6	75.0	80.0	55.0	80%
(\$/MWh)	6	41.1	79.5	80.0	56.6	98%
Dispatch	5	36.0	28.8	16.4	31.6 [3]	--
(MW)	6	36.0	36.8	49.1	40.4 [3]	--
	Total	72.0	65.6	65.5	72.0	

[1] Offers for Generators 5 and 6 set at \$80/MWh
[2] Offers for Generators 5 and 6 set at \$0/MWh
[3] Fixed to sum to 110% of the observed dispatch

Table 4.5 Sensitivity Analysis for Generators 5 and 6: Experiment II

	Generator	Competitive	Actual	High Offer[1]	Low Offer[2]	RMP
Price	5	55.0	72.0	80.0	51.7	72%
(\$/MWh)	6	54.3	70.0	79.0	50.5	68%
Dispatch	5	17.6	14.9	12.0	16.4[3]	--
(MW)	6	12.0	14.6	17.5	16.0[3]	--
	Total	29.6	29.5	29.5	32.4	

[1] Offers for Generators 5 and 6 set at \$80/MWh
[2] Offers for Generators 5 and 6 set at \$0/MWh
[3] Fixed to sum to 110% of the observed dispatch

For Experiment II, the total dispatch from generators 5 and 6 in the Competitive Solution, the Actual Experiment and the High Offer cases are all close to 29.5MW. Once again market power is not a local phenomenon. Since observed prices are lower than the maximum levels, the full potential for market power has not been exploited as effectively as it was in Experiment I.

The RMP is proposed in Section 4.1 as a measure of exploitation and it measures the percentage of the difference between the reservation price and the zero offer price that is actually observed. The values of RMP for Experiments I and II are shown in Tables 4.4 and 4.5 for generators 5 and 6. These results show that the participants in Experiment I were more successful in exploiting market power than the participants in Experiment II. This conclusion agrees with the discussion in Section 3. Exploiting monopoly power required tacit collusion in Experiment II, and the results in Figure 3.4 show that prices for generators 5 and 6 are still trending upwards after 75 rounds of the auction. The situation for Experiment I is different because both generators 5 and 6 are essential to meet load. This fact can be determined directly by noting that the total dispatch for generators 5 and 6 in the Actual Experiment and the High Offer case is greater than the total capacity of 60MW owned by each generator.

5. Conclusions

The primary objective of this paper is to show how the information contained in an Optimal Power Flow (OPF) for a power system can be converted into useful indicators of market power. The basic concept is that generators can be viewed as inputs into a production process to meet a pattern of loads (outputs) on a network. The optimal nodal prices, using a last accepted offer auction, correspond to input prices. The OPF solution corresponds to minimizing total cost subject to operating constraints. The first order derivatives of minimum cost with respect to nodal prices give the demand for generation at different nodes. The second order derivatives give the complete matrix of price effects for all generators.

The price effects are used to determine price elasticities at the solution values of the OPF. These elasticities are then combined to calculate a sufficient condition for the existence of market power that does not depend on knowing the true costs of production (see Section 2). The basic measure is that a weighted sum of the own-price and cross-price elasticities for a group of generators should be greater than -1.

Section 3 describes two different experiments of market power using POWERWEB to simulate an AC network with six generators. Students and utility executives act as generators in a uniform price auction to sell power. These experiments show that two of the generators are able to discover that they possess market power and use this knowledge to raise prices above competitive levels. In Section 4, the proposed measures of market power are applied to the experimental results to demonstrate that the measures can identify the existence of market power correctly.

An additional objective of Section 4 is to measure how well market power has been exploited. A measure of Relative Market Power is proposed which does distinguish correctly between the two experiments. However, this measure of exploitation is dependent on using the marginal cost of importing additional power as a proxy for knowing the true production costs. Our overall conclusion is that measuring the existence of market power in a network using the information in an OPF is a potentially valuable tool for an ISO to learn more about the economic characteristics of a power system. However, more research is needed to determine if better measures of exploitation can be developed. The implications of withholding capacity from an auction is a specific area that should be investigated further.

In a real auction, an ISO has more information than the optimal dispatch and nodal prices from an OPF. Actual offers can be compared to nodal prices, and in many cases, nodal prices may be substantially higher than the offers. For example, Eastern Electricity was one of the companies in the UK market that benefited from the high prices set by the dominant duopoly, by acquiring capacity and selling as much as possible (see Bower and Bunn (1999)). The ISO also has information about the total capacity offered into the auction. Assuming that all generators must disclose their rated generating capacities to an ISO, it would be possible to identify cases in which capacity inside a load pocket with high nodal prices is withheld from the auction. In the two experiments discussed in this paper, withholding capacity from the auction did not occur. However, other experiments have shown that this type of behavior is important for generating price spikes in a uniform price auction when load is stochastic. Our current research plan includes investigating the implications of load uncertainty on measurements of market power.

6. Bibliography

Borenstein, Severin, and James Bushnell (1997). "An Empirical Analysis of the Potential for Market Power in California’s Electricity Industry," University of California Energy Institute, Berkeley.

Borenstein, Severin, James Bushnell and Christopher Knittel (1999). “Market Power in Electricity Markets: Beyond Concentration Measures,” *The Energy Journal* 20(4): 65-88.

Borenstein, Severin, James Bushnell, and Frank Wolak (2000). "Diagnosing Market Power In California's Deregulated Wholesale Electricity Market.” University of California Energy Institute Power Working Paper 064, July.

Bower, John, and Derek Bunn (1999). "A Model-Based Comparison of Pool and Bilateral Market Mechanisms for Electricity Trading." London Business School Energy Markets Group.

Bunn, Derek W. (1999). “Electricity Markets are Different,” Strategic Price Risk in Wholesale Power Markets,” London Business School.

Chambers, Robert G. (1988). *Applied Production Analysis: A Dual Approach,* Cambridge: Cambridge University Press.

Ede, Simon, Timothy D. Mount, William Schulze, Richard Schuler, Robert Thomas, and Ray Zimmerman (2000). "Experimental Tests of Deregulated Markets for Electric Power: Market Power and Self Commitment," Report to the U.S. Department of Energy.

Federal Energy Regulatory Commission (2000). Commission Proposes to Reshape California’s ‘Seriously Flawed’ Electricity Markets with Sweeping Changes,” *News Release*, November, Washington, D.C.

Fuss, Melvyn and Daniel McFadden (eds.) (1978). *Production Economics: A Dual Approach to Theory and Applications, Vol. 1,* Amsterdam: North-Holland Publishing Company.

Hogan, William W. (1997). “A Market Power Model with Strategic Interaction in Electricity Networks,” Center for Business and Government, John F. Kennedy School of Government, Harvard University, Cambridge MA.

Joskow, Paul L. and Jean Tirole (2000). “Transmission Rights and Market Power on Electric Power Networks,” *Rand Journal of Economics* 31(3): Autumn.

Newbery, David (1995). "Power Markets and Market Power," *Energy Journal* 16(3): 39-66.

Oppel, Richard A. Jr. and Lowell Bergman (2001). "Deal for Use of Gas Pipeline Stirs Dispute on Competition," *The New York Times* March 26, 2001.

Pirrong, Craig (2000). *Manipulation of Power Markets,* John M. Olin School of Business, Washington University, St. Louis.

Rudkevich, Aleksandr, Max Duckworth and Richard Rosen (1998). “Modeling Electricity Pricing in a Deregulated Generation Industry: The Potential for Oligopoly Pricing in a Poolco,” *The Energy Journal,* 19(3): 19-48.

Schweppe, Fred C., Michael C. Caramanis, Richard D. Tabors and Roger E. Bohn (1988). *Spot Pricing of Electricity,* Boston, MA: Kluwer Academic Publishers.

Varian, Hall R. (1978). Microeconomic Analysis, New York: W.W. Norton & Company.

von der Fehr, Nils-Henrik Mørch and David Harbord (1993). "Spot Market Competition in the UK Electricity Industry," *The Economic Journal* 103(418): 531-546.

Weber, James D. and Thomas J. Overbye (2000). "An Individual Welfare Maximization Algorithm for Electricity Markets," submitted for publication in *IEEE Transactions on Power Systems* February 2000.

Wolak, Frank A. and Robert H. Patrick (1997). The Impact of Market Rules and Market Structure on the Price Determination Process in the England and Wales Electricity Market, University of California Energy Institute PWP 047.

Wolak, Frank A., Robert Nordhaus, and Carl Shapiro (2000). "An Analysis of the June 2000 Price Spikes in the California ISO's Energy and Ancillary Services Market," Market Surveillance Committee, California Independent System Operator, Sacramento.

Wolfram, Catherine D. (1999). Measuring Duopoly Power in the British Electricity Spot Market. *American Economic Review* 89:4: 805-827.

Appendix A. Deriving the Hessian Matrix of Price Effects

This appendix shows how to compute the sensitivity of dispatch to nodal prices starting from the solution to a general nonlinear optimal power flow problem. This is defined as the mathematical program $\min f(x)$ subject to $g(x) = 0$.

For convenience, x is assumed to have the following structure:

$$x = \begin{pmatrix} u \\ y \end{pmatrix}$$

where u are the real power injections that turn out to be free (i.e., not against a minimum or maximum operating limit), and y is everything else. This includes real power injections that are fixed due to being against a physical limit at optimality, as well as all reactive injections, bus voltage magnitudes, bus voltage angles and any other control variables or state variables.

Similarly, g has the structure

$$g(x) = \begin{pmatrix} g_1(x) \\ g_2(x) \end{pmatrix}$$

with g_1 being the load flow equations describing the real power mismatch exactly at the buses where the u variables apply, i.e.

$$g_1^i(x) = \sum_{k \in Bi} G^{ik} V^i V^k \cos(\theta^i - \theta^k) + \sum_{k \in Bi} B^{ik} V^i V^k \sin(\theta^i - \theta^k) - u^i$$

where B_i is the set of indexes of buses that are connected to the ith bus by means of an electrical branch, V^j and _^{j} stand for the voltage magnitude and voltage angle at the jth bus, and G^{ik}, B^{ik} are the real and imaginary part of the bus nodal admittance matrix element related to the interconnection of the ith and kth buses. The vector $g_2(x)$ is made of all other constraints in the problem, namely, real power mismatch at all other buses, reactive mismatch at all buses, line and transformer thermal limits, voltage limits, generation limits and any other binding constraint in the formulation.

The Lagrangian for this problem is

$$L(x, \lambda) = f(x) + \lambda^T g(x)$$

where the vector of Lagrange multipliers λ is partitioned as $\lambda = (\lambda_1, \lambda_2)$, in a manner commensurate with $g = (g_1, g_2)$. The first order optimality conditions are:

$$\nabla f(x) + \left(\frac{\partial g(x)}{\partial x}\right)^T \lambda = 0$$
$$g(x) = 0$$

Close to a solution, values of (x, λ) that are also solutions for a perturbed problem must satisfy

$$\begin{pmatrix} \lambda_1 \\ 0 \end{pmatrix} + \left(\frac{\partial g(x)}{\partial x}\right)^T \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = 0$$
$$g(x) = 0$$

and indeed, any $(\Delta x, \Delta\lambda)$ must satisfy the first order expansion of the above, which, for the case when the cost functions are linear, takes the form:

$$\left(\sum \lambda^i \nabla^2 g^i(x)\right)\Delta x + \left[\begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix} + \left(\frac{\partial g(x)}{\partial x}\right)^T\right]\begin{pmatrix} \Delta\lambda_1 \\ \Delta\lambda_2 \end{pmatrix} = 0$$
$$\frac{\partial g(x)}{\partial x}\Delta x = 0$$

Several simplifications are possible; first, notice that

$$\frac{\partial g_1}{\partial u} = -I\ .$$

Also,

$$\frac{\partial g_2}{\partial u} = 0$$

because the other active mismatches, reactive mismatches or line limits are not in terms of the free active injections. Finally, note that in the weighted Hessian sum

$$\sum \lambda^i \nabla^2 g^i(x) = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}$$

only the H_{22} term is nonzero, since the *u* variables enter *g* affinely. Thus, the above equations simplify to

$$H_{22}\Delta y = -\frac{\partial g_1}{\partial y}^T \Delta\lambda_1 - \frac{\partial g_2}{\partial y}^T \Delta\lambda_2 \qquad (1)$$

$$\Delta u = \frac{\partial g_1}{\partial y}\Delta y \qquad (2)$$

$$\frac{\partial g_2}{\partial y}\Delta y = 0 \qquad (3)$$

From (1)

$$\Delta y = -H_{22}^{-1}\left(\frac{\partial g_1}{\partial y}^T \Delta\lambda_1 + \frac{\partial g_2}{\partial y}^T \Delta\lambda_2\right) \tag{4}$$

This can be substituted into (3), which can then be solved for $\Delta\lambda_2$:

$$\Delta\lambda_2 = -\left[\frac{\partial g_2}{\partial y} H_{22}^{-1} \frac{\partial g_2}{\partial y}^T\right]^{-1} \frac{\partial g_2}{\partial y} H_{22}^{-1} \frac{\partial g_1}{\partial y}^T \Delta\lambda_1$$

This can be substituted into (4) and the result substituted in (2), to give the following expression for the sensitivity of dispatch to price:

$$\Delta u = \frac{\partial g_1}{\partial y}\left\{H_{22}^{-1} \frac{\partial g_2}{\partial y}^T \left[\frac{\partial g_2}{\partial y} H_{22}^{-1} \frac{\partial g_2}{\partial y}^T\right]^{-1} \frac{\partial g_2}{\partial y} H_{22}^{-1} - H_{22}^{-1}\right\}\frac{\partial g_1}{\partial y}^T \Delta\lambda_1 = M\Delta\lambda_1$$

where M is the symmetric matrix of price effects discussed in Section 2.
Given a solution to an optimal power flow problem, it is possible to compute the sensitivity matrix directly with this formula for small to medium-sized systems. For larger systems, the sparsity of H_{22} must be exploited.

Appendix B

The POWERWEB Platform

POWERWEB is designed to be a flexible Internet-based platform for performing economic experiments. To date the experiments implemented using this platform have focused on examining the behavior of electricity markets using realistic modeling of the physical transmission network and real human decision-makers. Its Internet-based architecture eliminates the need for participants to be physically present in a specially equipped laboratory. The POWERWEB server handles application logic, data processing and computation. Users submit offers to sell power through a standard web browser.

In the electricity markets currently implemented in POWERWEB, each participant in a session plays the role of an owner of a generating plant offering to sell power through an independent system operator (ISO). An example offer submission page is shown in Figure A-1.

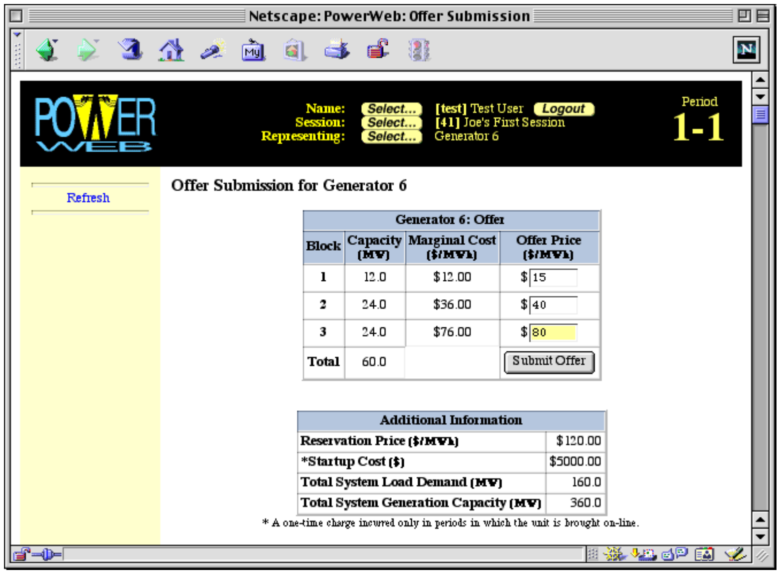

Figure B-1: Offer Submission Page from PowerWeb

For most experiments, POWERWEB selects the successful offers from competing generators through a uniform price last accepted offer auction. It produces, via an optimal power flow simulation, the market clearing prices and the generation schedules which optimally meet demand (while respecting all of the physical constraints of the power system). The page shown in Figure B-2 displays the results of a single auction, and the selling prices correspond to nodal pricing. For the example shown in Figure B-2, generator 6 sells the first two blocks of capacity at the same price of $4.03/MWh, having submitted offers of $15/MWh and $40/MWh, respectively.

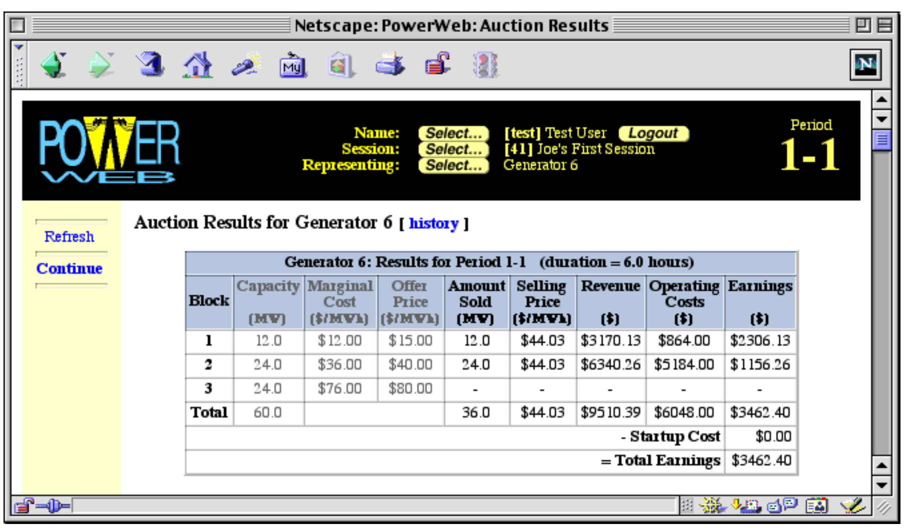

Figure B-2: Auction Results Page from POWERWEB

Figure B-3 is a diagram of a 30 bus, 6 generator power system whose (some 200) physical characteristics and constraints are modeled by POWERWEB's "smart market". An important feature of POWERWEB is that a full AC network is simulated. This is essential for studying market power, because the ability of a generator to support voltage at a particular location is often a binding constraint when transmission lines are fully loaded. The current network in POWERWEB was derived from a simplified representation of the New England Power Pool. However, the live constraints and other features are modified to create the appropriate conditions for an experiment.

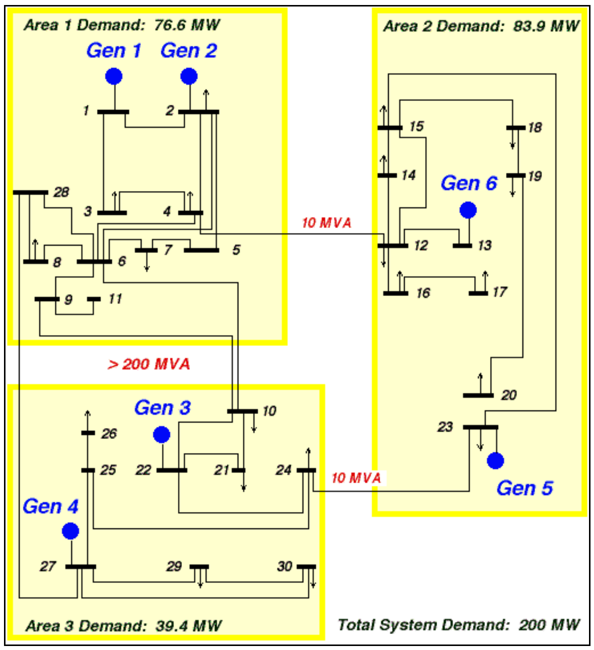

Figure B-3: The Underlying Network for the Auction