# An Agent-based Optimal Bidding Function

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## Abstract

*Problems such as price volatility have been observed in electric power markets. Demand-side participation is often offered as a potential solution by promising to increase market efficiency when hockey-stick type offer curves are present. However, individual end-consumer will surely value electricity differently, which makes demand-side participation as a group and at a bus difficult. In this paper demand is categorized into two groups: one that highly values reliability and the other that does not. The two types are modeled separately and a new optimal bidding function is developed and tested based on this model.*

## I. Introduction

In several of the deregulated electricity markets the demand-side is treated as inelastic. In reality, the demand-side is capable of making rational decisions about its need to serve or not serve load at any price. In current retail electricity markets consumers rarely buy electric energy at spot prices [1]**.** Consequently, most consumers do not have a financial incentive to reduce the price they pay for energy. Some researchers claim that demand-side participation should promote market efficiency [1 – 7]. In this case large consumers would participate by submitting bids directly into the market while small consumers could band together through an aggregator. For most current market structures a distributor may play the role of an aggregator in order to buy electric power from suppliers. They would then distribute electric power based on various pre-determined priorities.

For practical reasons, any new method for demand-side participation should include the following features; 1) the process should not be too time consuming, 2) a small discrepancy between the actual and fulfilled demand is not a significant matter for most consumers, 3) some consumers are willing to sacrifice reliability to reduce their price for electricity. This is referred to as price-based demand and 4) others need a high degree of reliability and are willing to pay for it. This is termed must-serve demand.

In order to develop a demand-side model we assume consumers are willing to marginally sacrifice reliability and frequently the use of price-responsive-load (PRL) should be avoided. At the end of each period an aggregator could check to see if there is a discrepancy between the dispatch slated to satisfy their demand and the actual demand they contracted for. If the discrepancy does not exceed a predetermined value, the aggregator can assume all of the demand was met. Otherwise, the aggregator would declare the need for a PRL, that is, the need to curtail some load. Suppose for example that contractually a PRL can be declared at most a fixed number of times per month. That is, for this fixed number of times per month an aggregator is not required to satisfy all of its requested demand. Consequently, the aggregator has a freedom to not satisfy all demand that is proportional to the remaining allowed number of PRL's for that month. The freedom is inversely proportional to the number of remaining periods before the current contract period ends since there is a non-negligible possibility of having to declare a PRL for more periods than remain on the contract.

An aggregator or demand-side agent could enter into price-based demand contracts with its many end consumers that contain following terms; 1) there are $n_{PRL}$-times a PRL is allowed during a given contract period where the PRL term or length is defined to be the case where less than a predetermined percentage of the forecast and/or actual demand is served, 2) after PRL's have been called $n_{PRL}$-times in the given period the bid should approach an inelastic demand curve, and finally 3) a substantial penalty should be assessed if the agent does not serve the agreed upon demand more than $n_{PRL}$-times when it could. An agent might offer several options for different values of $n_{PRL}$ to accommodate the various needs of the consumers.

For a must-serve type demand it is possible to have another type of contract; one that allows a distributor to always serve a customer before serving any other demand if price falls below a certain price, $p_c$. Since the demand forecast is not always accurate and must-serve demands prefer to be served even when the forecast underestimates their demand, the contract should include another price for insurance. Note that the price for insurance should always be less than $p_c$. For example, a contract could be defined by constants

$\xi$ and $p_m$ such that the consumer agrees to pay for electricity for an actual demand up to ($1+\xi$) times the forecasted demand if the price is less than or equal to $p_m$.

## II. Demand-price modeling

In developing a demand model we assume there is a minimum quantity of energy needed to satisfy an individual load-serving-entity's (LSE) demand. Consequently, demand can be quantized and ranked in terms of a priority which is evaluated in terms of a bid price. Since elementary demand has the property that it is ranked by price and quantity, there exists a form of an exclusion principle for the case where available quantity is limited. When all the elementary demand needs are ordered according to their priorities, one can construct a bid function, i.e., a demand curve which represents a willingness-to-pay for electricity. Note that there is no limit to the available quantity for must-serve demand. Consequently, an exclusion principle does not apply to demand classified as must-serve.

Since a supply-side offer price increases with respect to the quantity offered, the quantities needed to fulfill demand occupy a different state according to their offered prices. The state is termed a quantity state. Suppose there are N total individual demands and assume each state has an allowed quantity state $q_i$ (MW) that can be used to to satisfy the total individual demand. Let $g_i$ be the capacity of the quantity state $q_i$, i.e., number of maximum demand satisfied at quantity state $q_i$ and $n_i$ be the actual number of demands fulfilled. Note that the values of $q_i$ and $g_i$ are fixed, and the value of $n_i$ is the solution to an optimization problem and is between zero and $g_i$.

From a thermodynamic theory, a system is most stable when its entropy is maximized. The entropy is approximately evaluated in terms of the number of possible configurations as follows [8]:

$$S = k \log W \tag{1}$$

where S, k and W stand for system entropy, the Boltzmann constant and the number of possible configurations, respectively.

For a demand-side agent, the most stable (and therefore optimal) configuration corresponds to a distribution of fulfilled demands that maximizes demand-side profits. The bid function describes the optimal distribution. This distribution must satisfy two constraints: 1) the number of total fulfilled demands should be no more than that of the total demands, and 2) the total quantity state for fulfilling demand should be no more than the quantity state defined by the offered quantity. With given setup, one wants to optimize the distribution of fulfilled demand based on preferences. That is, solve the optimization problem:

$$\begin{aligned} &\max_{n_i} \; \mathrm{k} \log W(n_i) \\ &\text{subject to} \quad \sum_i n_i \le N_{tot} \\ &\qquad\qquad\quad \sum_i n_i q_i \le Q_{tot} \end{aligned} \tag{2}$$

where $N_{tot}$ is the total number of individual demands and $Q_{tot}$ is the total quantity state. Note that the Boltzmann constant shown in equation (1) is dropped for sake of simplicity since it is a positive constant of only historical significance.

### II-I. Price-based demand

In general a distributor is indifferent to which individual demand is being served since individual demand is not separable at the bus level. For example, suppose there are two individual demands and each need 3 MW. If only 3MW's are available, then only one demand can be fulfilled, i.e., partial fulfillment of both demands is not possible. Therefore, for a demand fulfilling state $n = (n_1, n_2, n_3, ...)$ where the demand in the parenthesis are the fulfilled demand to be determined. The conditional probability that a state $n_i$ given quantity $q_i$ to be fulfilled is $f(q_i \mid n) = n_i / g_i$. Then, the distribution $f(q_i)$ is given by:

$$\begin{aligned} f(q_i) &= \sum_n f(q_i \mid n) p(n) = \sum_n f(q_i \mid n) \frac{W(n)}{W_{tot}} \\ &= \frac{1}{W_{tot}} \sum_n f(q_i \mid n) W(n) \end{aligned} \tag{3}$$

where $W(n)$ is the number of demands fulfilled and $W_{tot}$ is the total possible number of demands fulfilled.

To calculate the distribution $W(n)$, count the number of combinations it takes to place $n_i$ fulfilled demand into $g_i$ offered quantity state when $g_i > n_i$. Note that the number of total offered quantity states is the total capacity needed to accommodate the requested demand. Since the number of fulfilled demands at the

$i^{th}$ quantity state cannot exceed the capacity, the $i^{th}$ quantity state should contain both occupied and unoccupied demand. The number of possible combinations of those occupied and unoccupied states is:

$$w(n_i) = \binom{g_i}{n_i} = \frac{g_i!}{(g_i - n_i)!n_i!} \quad (4)$$

Since each combination for $n_i$ in the distribution $n$ is independent, the overall distribution for $n$ states $W(n)$ is

$$W(n) = \prod_i w(n_i) = \prod_i \frac{g_i!}{(g_i - n_i)!n_i!} \quad (5)$$

Equation (5) can be evaluated by using Stirling approximation for a sufficiently large N.

$$\begin{aligned} &\max_{n_i} \log W(n_i) \\ &\text{subject to } \sum_i n_i \le N_{tot} \\ &\qquad \sum_i n_i q_i \le Q_{tot} \end{aligned} \quad (6)$$

Forming the Lagrangian for the problem yields

$$\begin{aligned} L(n) &= \log W(n) + \mu_1\left(N_{tot} - \sum_i n_i\right) \\ &\quad + \mu_2\left(Q_{tot} - \sum_i n_i q_i\right) \\ &= \sum_i \{\log(g_i!) - \log[(g_i - n_i)!] - \log(n_i!)\} \\ &\quad + \mu_1\left(N_{tot} - \sum_i n_i\right) + \mu_2\left(Q_{tot} - \sum_i n_i q_i\right) \end{aligned} \quad (7)$$

The Lagrangian can be approximated by using the Stirling formula for approximating the log of a factorial:

$$\begin{aligned} L(n) &= \sum_i [g_i \log g_i - (g_i - n_i)\log(g_i - n_i) \\ &\qquad n_i \log n_i] \\ &\quad + \mu_1\left(N_{tot} - \sum_i n_i\right) + \mu_2\left(Q_{tot} - \sum_i n_i q_i\right) \end{aligned} \quad (8)$$

The Kuhn-Tucker multipliers ($\mu_1$ and $\mu_2$) and the critical points for individual demand are found from the necessary conditions for a maximum. These Kuhn-Tucker conditions [9] are

$$\begin{aligned} &n_i\left[\frac{\partial L(\vec{n})}{\partial n_i}\right] = n_i\left[\log\left(\frac{g_i - n_i}{n_i}\right) - \mu_1 - \mu_2 q_i\right] = 0 \\ &\mu_1\left(N_{tot} - \sum_i n_i\right) = 0 \\ &\mu_2\left(Q_{tot} - \sum_i n_i q_i\right) = 0 \end{aligned} \quad (9)$$

Since fulfilled demand $n_i$ is nonzero by definition, the first part of equation (9) yields

$$\frac{n_i}{g_i} \equiv f(q_i) = \frac{1}{1 + \exp(\mu_1 + \mu_2 q_i)} \quad (10)$$

The distribution described in equation (10) optimizes the fulfilled demand profile.
The Kuhn-Tucker multipliers are the shadow prices of the corresponding constraints. Fulfilling one additional demand increases the demand-side profit while requiring more electricity might increase the market clearing price. In other word, the addition of one more unit of demand increases the value of the Lagrangian by $\mu_1$ if the constraint is binding, but an increase in demand reduces the profits of the demand-side by requiring more electricity. Consequently, $\mu_1$ takes a negative value. On the other hand, adding one additional quantity to the system increases profit of demand-side by $\mu_2$. This addition results in different satisfaction to individual demand since $i^{th}$ demand needs $q_i$ quantity to fulfill it. From a demand-side perspective, market clearing prices tend to be low when more electricity is available, which results in increasing demand-side profit. The Kuhn-Tucker multipliers are given by

$$\begin{aligned} \mu_1 &= -\frac{q_F}{fr} \\ \mu_2 &= \frac{1}{fr} \end{aligned} \quad (11)$$

where $q_F$ and fr represent a reference quantity and a freedom factor that describes the freedom that an agent has. The value of $\mu_1$ is negative with the magnitude of weighted reference quantity which individual demand has on average. $\mu_2$ is positive with a value of inverse of freedom that demand-side agent has. When a demand-side agent has more freedom, it does not add one more quantity, i.e., less value for $\mu_2$.

## II-II. Must-serve demand

When a demand must be served such as in hospital, synchrotron etc., a quantity state can accommodate as many demands as possible. In such a case, there is no limit to the occupation number at each level, i.e., no exclusion principle exists. For finding optimal fulfilling distribution for such demands, one calculates the number of ways to assign $n_i$ fulfilled demands in $q_i$ quantity states. Note that all the demands are indistinguishable to a demand-side agent. Since price is not important to fulfill such demands, all the quantity states are identical if the prices for the states are acceptable. Then all available state can be fulfilled regardless $q_i$. After fulfilling, one can find the fulfillment configuration by finding which state is occupied. Therefore the problem is distributing identical demands on various sites where there is no limit. In the case, the number of possible configurations is:

$$w(n_i) = \binom{n_i + g_i - 1}{n_i} = \frac{(n_i + g_i - 1)!}{n_i!(g_i - 1)!} \tag{12}$$

Then one can construct an optimization problem with the same constraints used in the previous case. Thus, the Lagrange method for the problem gives:

$$\begin{aligned} L(n) &= \log W(n) + \mu_1\left(N_{tot} - \sum_i n_i\right) \\ &\quad + \mu_2\left(Q_{tot} - \sum_i n_i q_i\right) \\ &= \sum_i \{\log[(n_i + g_i - 1)!] - \log(n_i!) \\ &\quad - \log[(g_i - 1)!]\} \\ &\quad + \mu_1\left(N_{tot} - \sum_i n_i\right) + \mu_2\left(Q_{tot} - \sum_i n_i q_i\right) \end{aligned} \tag{13}$$

Since $n_i$ and $g_i$ are sufficiently large, 1 in equation (13) can be ignored and applying Stirling approximation yields the Lagrangian approximated:

$$\begin{aligned} L(n) &= \sum_i [(n_i + g_i)\log(n_i + g_i) - n_i \log n_i \\ &\quad - g_i \log g_i] \\ &\quad + \mu_1\left(N_{tot} - \sum_i n_i\right) + \mu_2\left(Q_{tot} - \sum_i n_i q_i\right) \end{aligned} \tag{14}$$

For maximization, one needs to find optimality conditions for individual demand as well as Lagrange multipliers:

$$\begin{aligned} & n_i\left[\frac{\partial L(\vec{n})}{\partial n_i}\right] = n_i\left[\log\left(\frac{n_i + g_i}{n_i}\right) - \mu_1 - \mu_2 q_i\right] = 0 \\ & \mu_1\left(N_{tot} - \sum_i n_i\right) = 0 \\ & \mu_2\left(q_{tot} - \sum_i n_i q_i\right) = 0 \end{aligned} \tag{15}$$

As was mentioned earlier, $n_i$ cannot be zero. Then, the first part in equation (15) yield

$$\frac{n_i}{g_i} \equiv f(q_i) = \frac{1}{\exp(\mu_1 + \mu_2 q_i) - 1} \tag{16}$$

Note that the Lagrange multipliers are associated with the same constraints as before.

## III. Optimal bidding functions

The bidding function is the distribution function that optimizes demand-side satisfaction. For an agent, priority of an individual demand is important only when it reflects true evaluation in terms of bidding price, i.e., $B = p/p_{max}$ where B, p and $p_{max}$ represent a priority of each demand, bidding price and the maximum possible bidding price for a current period, respectively. An agent needs to get electric power to meet demand. For example, suppose an agent has five more remaining periods before contract for price-based demand expires, but it has six more allowed PRL's In such a case, the agent may not need to satisfy all the demands. Then freedom, fr, can be defined

$$fr = \begin{cases} m\dfrac{n}{N} & \text{if } n < N \\ \infty & \text{if } n \geq N \end{cases} \tag{17}$$

where m is a positive constant, and n and N stands for the number of allowed PRL remained and remaining periods for next bid, respectively.

One unaddressed subject is the reference quantity, $q_F$. Every period, an agent is informed a demand forecast from ISO. By using a forecast $q_f$, a bidding function can be written;

$$p(q) = \frac{p_{max}}{1 + r\exp\left(\dfrac{q - q_f}{fr}\right)} \tag{18}$$

where $r \equiv \exp\left(\frac{q_f - q_F}{fr}\right)$.

To evaluate r, consider the two following extreme cases; 1) an agent has no freedom, i.e., the agent exhausted all allowed PRL's before current contract expires, and 2) an agent has infinite freedom i.e., the number of unused PRL's is greater than that of remaining periods. For the first case, $fr = \mathrm{m}(n/N)\big|_{n=0} = 0$ which leads the following equation, which explains the reason an agent must accept any price because it cannot afford any more PRL's:

$$p(q) = \left.\frac{p_{\max}}{1 + r\exp\left(\frac{q - q_f}{fr}\right)}\right|_{fr\to 0} = p_{\max} u\left(q - q_f\right) \quad (19)$$

where u is the unit step function:

$$u(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{otherwise} \end{cases} \quad (20)$$

For the second case, an agent offers a reasonable price, $p_r$, for all the quantities, i.e.:

$$p(q) = \left.\frac{p_{\max}}{1 + r\exp\left(\frac{q - q_f}{fr}\right)}\right|_{fr\to\infty} = \frac{p_{\max}}{1 + r} = p_r \quad (21)$$

which leads $r = \frac{p_{\max} - p_r}{p_r}$. Therefore, an optimal bidding function for a price-based demand is written

$$p(q) = \frac{p_{\max}}{1 + \frac{p_{\max} - p_r}{p_r}\exp\left(\frac{q - q_f}{m\frac{n}{N}}\right)} \quad (22)$$

For must-serve demand, a bidding function can be written for a given forecast $q_f$

$$p(q) = \frac{p_{\max}}{h\exp\left(\frac{q - q_f}{fr}\right) - 1} \quad (23)$$

where $h \equiv \exp\left(\frac{q_f - q_F}{fr}\right)$.

To evaluate h, consider the prices for the quantity at $q = q_f$ and at $q = (1+\xi)q_f$, which was promised to the consumer by contract. For the first case, $p = p_c$ which leads to the following equation.

$$p(q) = \left.\frac{p_{\max}}{h\exp\left(\frac{q - q_f}{fr}\right) - 1}\right|_{q=q_f} = \frac{p_{\max}}{h - 1} = p_c \quad (24)$$

which leads to $h = \frac{p_{\max} + p_c}{p_c}$

For the second case, the offer price should be $p_m$ according to the contract, i.e.:

$$p(q) = \left.\frac{p_{\max}}{h\exp\left(\frac{q - q_f}{fr}\right) - 1}\right|_{q=(1+\xi)q_f} = \frac{p_{\max}}{\frac{p_{\max} + p_c}{p_c}\exp\left(\frac{\xi q_f}{fr}\right) - 1} = p_m \quad (25)$$

which leads to

$$fr = \frac{\xi q_f}{\ln\left[\left(\frac{p_{\max} + p_m}{p_m}\right)\left(\frac{p_c}{p_{\max} + p_c}\right)\right]} = \frac{\xi q_f}{\eta} \quad (26)$$

where

$$\eta = \ln\left[\left(\frac{p_{\max} + p_m}{p_m}\right)\left(\frac{p_c}{p_{\max} + p_c}\right)\right] \quad (27)$$

Note that $\eta > 0$ since $p_c > p_m$. Consequently, a bidding function can be written as:

$$p(q) = \frac{p_{\max}}{\frac{p_{\max} + p_c}{p_c} \exp\left[\frac{\eta}{\xi}\left(\frac{q - q_f}{q_f}\right)\right] - 1} \qquad (28)$$

The shape of the bidding function, equation (28), is very steep curve similar to an inelastic demand curve. This can be understood since the demand is must-be-served. Fig. 1 illustrates the shapes of two demand bidding curves.

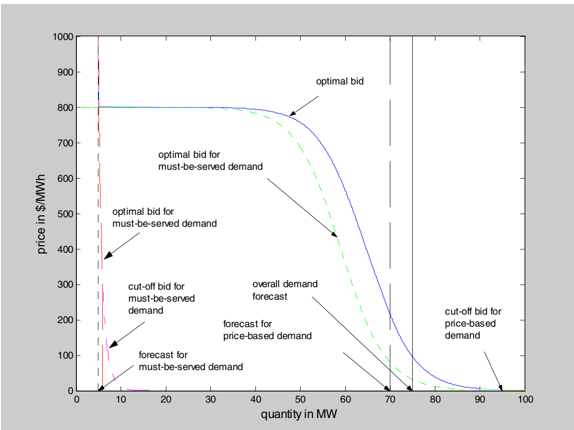


**Figure 1. Optimal bidding curves for maximizing the profit of consumers: Red and green lines correspond to an optimal bids for must-serveand price-based demands, respectively. Overall optimal bid at given forecasts for must-serveand price-based demand follows blue line.**

As the freedom fr approaches to zero, the values for bidding prices for both types of demand go to maximum price since the demand-side agent must fulfill all demands including price-based demand. At a given price p, an agent would like to purchase $q_m$ and $q_p$ to fulfill must-served-demand and price-based demand, respectively. Consequently, an agent would purchase sum of the quantities, i.e., $q_m + q_p$ at the price p. Since demand forecast has an error, the agent might purchase more than forecast by a certain amount. However, there is no need for an agent to bid for a quantity more than needed, and then the bidding price drops to zero, which is termed cut-off bid.

Fig. 2 shows bidding curves for several cases. Blue line stands for the bid curve submitted at $n^{th}$ period. If there was no PRL at the period, freedom increases only slightly from n/N to n/(N-1). Consequently, bid price is reduced in a very small amount. On the other hand, an occurrence of PRL significantly reduces the value of freedom, and then bid curve becomes much steeper.

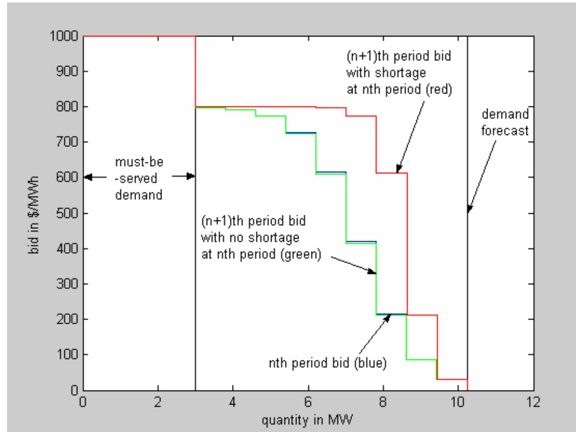


**Figure 2. Change in bidding after one period subject to the occurrence of an energy PRL. Optimal bid at a period depends on the results from previous periods according to contracts between an agent and end-consumers**

## IV. Simulation results and discussion

A modified IEEE 30 bus system with 6 generators and 20 loads was used for a simulation. Fig. 3 shows the transmission network used in this study. In the system, there are three different areas divided due to line constraints. Due to strict line constraints between Area 2 and the rest of the system, Firm 5 and 6 have a locational benefit in case of heavy load in the Area 2, which implies a potentially duopoly situation.

Actual demand seems to have weekly and hourly periodicities. To mimic these periodic behaviors, a convolution between two sine functions added with small values of random number was taken for an actual demand. For the purpose, convolution of two functions f(x) and g(x) over a finite range [0, t] is performed in the following way:

$$f \otimes g = \int_0^t f(\tau) g(t - \tau) d\tau \qquad (29)$$

where the symbol $f \otimes g$ denotes convolution of f and g.

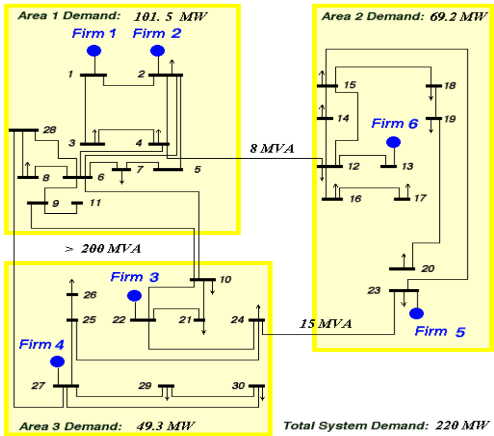


**Figure 3. Modified IEEE 30 bus system with six suppliers. Capacities of lines connecting Area 1–Area 2 and Area 3–Area 2 are lower than those of other lines.**

To emulate the periodic behaviors of load as well as stochastic behaviors, the functions f and g were taken as

$$
\begin{aligned}
f(t) &= \frac{A_{day}}{2}\left[\sin\left(\frac{2\pi}{\tau_{day}}t\right)+1\right]+B_{day}\delta_{day} \\
g(t) &= \frac{A_{week}}{2}\left[\sin\left(\frac{2\pi}{\tau_{week}}t\right)+1\right]+B_{week}\delta_{week}
\end{aligned}
\qquad (30)
$$

where A, B denote fluctuation of load for periodic part and that for stochastic part, respectively, and τ and δ stand for periodicity and for random value in [0, 1], respectively, and subscripts represent time period. Several simulations with various types of supply-side agents have been performed. The average prices from the simulations are plotted in Fig. 4. For Case a) which is the more competitive market, while the average market clearing price is about $ 550/MWh with inelastic demand without demand-side participation, it was less than $ 100/MWh with a demand-side participation. Case b) is more interesting in that there were many price spikes to meet inelastic demand due to highly volatile market. Demand-side participation lowers the number of price spikes as well as average price in both cases. Some price spikes are eliminated by backing up less than 10 % of load while some are reduced by a significant amount that requires declaring PRL. When significant deduction occurs, the number of allowed PRL is decreased by one which resulted in a decrease in freedom. Consequently, a bid curve approaches the inelastic demand curve. When number of allowed PRL goes to zero, the bid curve of demand-side agent looks identical to that of inelastic demand. Therefore, market clearing price are same. For example, the results with elastic demand after 720 periods are equal to those without.

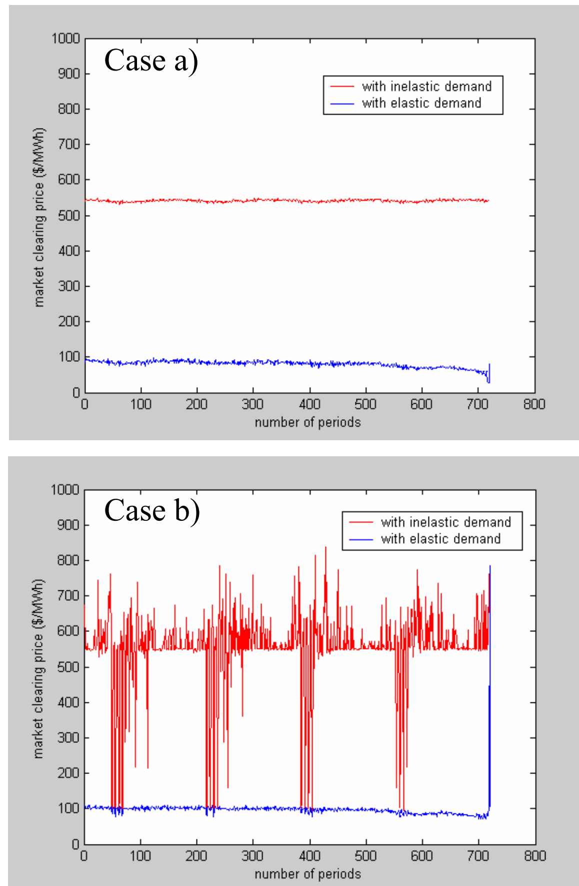


**Figure 4. Two typical simulation results with inelastic demand without demand-side participation (red line) and with elastic demand with demand-side participation (blue line). Case a) shows the results in a more competitive market than that for Case b).**

## V. Conclusion

Demand-side participation would help electricity power markets to operate in a more efficient way. Optimal bidding functions are theoretically derived by taking advantage of the new types of contracts being offered by electricity retailers. Then a feasible implementation suited for a retail market is presented. The proposed method is implemented to a modified

IEEE 30 bus system to show how such a demand-side participation enhance the market efficiency.

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